Auc2Charge: An Online Auction Framework for Electric Vehicle Park-and-Charge

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ABSTRACT

The increasing market share of electric vehicles (EVs) makes large-scale charging stations indispensable infrastructure for integrating EVs into the future smart grid. Thus their operation modes have drawn great attention from researchers. One promising mode called park-and-charge was recently proposed. It allows people to park their EVs at a parking lot, where EVs can get charged during the parking time. This mode has been experimented and demonstrated in small scale. However, the missing of an efficient market mechanism is an important gap preventing its large-scale deployment. Existing pricing policies, e.g., pay-by-use and flat-rate pricing, would jeopardize the efficiency of electricity allocation and the corresponding social welfare in the park-and-charge mode, and thus are inapplicable. To find an efficient mechanism, this paper explores the feasibility and benefits of utilizing auction mechanism in the EV park-and-charge mode. The auction allows EV users to submit and update bids on their charging demand to the charging station, which makes corresponding electricity allocation and pricing decisions. To this end, we propose *Auc2Charge*, an online auction framework. *Auc2Charge* is truthful and individual rational. Running in polynomial time, it provides an efficient electricity allocation for EV users with a close-form approximation ratio on system social welfare. Through both theoretical analysis and numerical simulation, we demonstrate the efficacy of *Auc2Charge* in terms of social welfare and user satisfaction.

Categories and Subject Descriptors

C.4 [**Performance of systems**]: Modeling techniques; J.7 [**Computers in other systems**]: Industrial control

Keywords

smart grid, electric vehicles, mechanism design, auction

1. INTRODUCTION

The electric vehicle (EV) is visioned as a crucial component in the future intelligent transportation systems (ITS) [4]. Compared with gasoline-powered vehicles, EVs have the potential benefits of a lower carbon emission, a lower powering cost and a higher power efficiency. With these promising benefits, however, EVs also introduce a high penetration into the power grid by shifting the energy load from gasoline to electricity. As EVs' market share is increasing, the integration of EV into the future smart grid has drawn great interests of both academia and industry. Various charging facilities have been studied [4, 8, 9, 12, 15, 17, 18].

Among all charging facilities (e.g., home charging point, workplace charging facility and etc.), charging stations have become indispensable infrastructure to support the large-scale development of EVs [9,10]. Thus their operation mode requires a careful design. Recently, researchers propose a promising operation mode of charging station, which is called EV park-and-charge. In this mode, people can park their EVs at the parking lot equipped with charging points. These vehicles can then be charged during the period of parking. Potential application scenarios of this mode include parking-lot charging at workplace, shopping mall, airport and military base. Several field experiments have been done to explore the feasibility of this operation mode. For instance, several universities in Europe conduct the V-Charge project, which aims to design an automated valet parking and charging system to support autonomous local transportation [3]. General Motors (GM) and TimberRock perform a pioneering experiment [2], in which TimberRock uses the OnStar vehicle communication system to manage the charging schedule of a fleet of Chevrolet Volts parked in GM's E-Motor Plant. Its objective is to balance the stochastic arrival of charging demand and the intermittency of renewable energy supply at the park-and-charge station. And the U.S. Air Force works with the Lawrence Berkeley National Laboratory to conduct a similar experiment at its Los Angeles Base [19]. Though these experiments provide positive feedback on the potential of the park-and-charge mode, one important gap still exists between small-scale experiments and the large-scale deployment of this mode. This gap is the missing of an efficient market mechanism.

An efficient market mechanism for charging station should achieve two objectives: 1)to avoid overpricing and underpricing the electricity by quickly adapting to the change in demand-supply relation; and 2)to provide an explicit guarantee on social welfare by constructing an efficient electricity allocation between EV users. The social welfare is the monetary sum of the revenue from charging station and the utility gained by EV users. Regardless the differences on hardware and charging schedule, current charging stations mainly adopt either one of the following pricing policies as their market mechanism: pay-by-use pricing and flat-rate pricing. Though they are simple and helpful for the market expanding of EVs, they are not efficient market mechanisms. For instance, overpricing and underpricing could happen in both policies due to the fluctuation of electricity price from power distributors, thus harm the benefits of EV users and charging station, respectively. What is worse, the long charging time of EV and limited capacity of charging station would cause inefficient electricity allocation between

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EV users and significantly prolong the total time (waiting time and actual charging time) spent by EV users. In extreme cases, some EVs would end up getting very little electricity charged after hours of waiting. And this inefficiency would become more severe as the market share of EV continues increasing.

To design an efficient market mechanism, we explore the feasibility and benefits of utilizing auction in the park-and-charge operation mode. In particular, we propose to design an online auction framework for this mode. Finding such an auction framework for this mode, however, is a non-trivial task because we need to address a series of challenges. 1)The proposed auction must be online to cope with the stochastic nature of system information in the park-and-charge mode. 2)The auction framework should be computationally efficient so that the electricity allocation and pricing decision can be quickly made in large-scale park-and-charge stations. 3)The auction framework should ensure truthfulness, individual rationality and an explicit guarantee on social welfare simultaneously.

To address these challenges, we propose *Auc2Charge*, a computationally efficient online auction framework, which leverages recent progress in mechanism design [5, 6, 16, 23]. It involves a budget-based bid update process between any consecutive two time slots [6, 23], which transforms the long-term total social welfare maximization problem(**PNC**) into a series of one-shot social welfare maximization problems $\mathbf{PNC_{one}(t)}$. For each $\mathbf{PNC_{one}(t)}$ problem, we design a greedy α-approximation algorithm using the classic primal-dual method [5], and translate it to a randomized one-shot auction mechanism, which is truthful and individual rational. It ensures an α-approximation ratio on social welfare of $\mathbf{PNC_{one}(t)}$ problem in polynomial time [16]. Different from auctions in cloud systems, where exists no constraint on users' capacity of receiving resources, we adopt a dropping process to drop bids violating maximal charging-capacity constraint in one-shot auction, without compromising social welfare. By integrating this one-shot auction with the budget-based bid update process, *Auc2Charge* provides an explicit approximation ratio on overall social welfare of park-and-charge while maintaining the property of truthfulness and individual rationality. Our **main contributions** are as follow:

1. We study the novel problem of utilizing auction in designing an efficient market mechanism for the EV park-and-charge mode. In particular, we propose *Auc2Charge*, a computationally-efficient online auction framework.

2. Through theoretical analysis, we show that *Auc2Charge* ensures the property of truthfulness and individual rationality, and provides a close-form competitive ratio on system social welfare.

3. Using numerical simulation, we demonstrate its efficacy under various settings in terms of social welfare and user satisfaction.

The remaining of this paper is organized as follows. In Section 2 we illustrate EV park-and-charge, discuss our motivation and identify the corresponding challenges. We present system settings and problem formulation in Section 3, and the design of *Auc2Charge* in Section 4. We study the performance of *Auc2Charge* in Section 5. We discuss related work in Section 6, and conclude our paper in Section 7.

2. PARK-AND-CHARGE: ILLUSTRATION, MOTIVATION AND CHALLENGES

In this section, we first illustrate the EV park-and-charge mode in Figure 1. In a large parking lot, every parking spot is equipped with a charging point. People drive to the parking lot, park their EVs, connect vehicles to charging points and leave for their personal arrangements, e.g., working, shopping and etc. During the parking period, charging points can charge electricity to the connected EVs. The detailed charging scheduling, e.g., charging speed and charging amount, is determined by a controller in the park lot.

The park-and-charge mode works in a dynamic environment, including fluctuate electricity supply, unpredictable arrival of EV charging demand and the ever-changing unit-time charging capacity of different EVs, all of which result in a stochastic demand-supply relation. To be deployed in large-scale, therefore, the park-and-charge mode needs an efficient market mechanism to quickly response to the change of demand-supply relation. However, existing pricing policies for charging stations, such as pay-per-use pricing and flat-rate pricing, fail to adapt to the rapid change of demand-supply relation in EV park-and-charge mode. For instance, when the available electricity supply is larger than the charging demand from parked EVs, both policies are overpricing electricity, which jeopardizes the benefit of EV users. On the contrary, when the available electricity supply is smaller than the charging demand from parked EVs, both policies are underpricing electricity as it is now a scarce resource. As a result, the benefit of charging station is jeopardized.

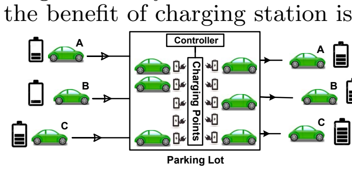
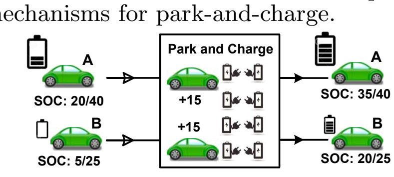
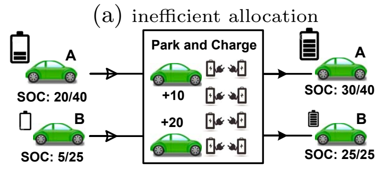


Figure 1: An overview of park-and-charge operation mode

What is worse, these pricing polices may yield an inefficient electricity allocation between EV users, and thus impair system social welfare, i.e., the monetary sum of revenue made by charging station and the utility gained by EV users. Take the scenario in Figure 2 as an example, where the park-and-charge station has an electricity supply of 30 kWh. When arriving at the parking-and-charge lot, EV user A has an SOC of 20kWh/40kWh=0.5 while user B has an SOC of 5kWh/25kWh=0.2. Only focusing on station's revenue, pay-per-use and flat-rate policies do not distinguish the utility difference under various allocation schemes as they yield the same revenue. Thus charging A and B by 15kWh each in Figure 2a) appears to be a fair allocation for these policies. However, the lower SOC of B, i.e., 0.2, implies that a unit amount of electricity would bring a higher utility to user B than A. To maximize total social welfare, an efficient electricity allocation should charge B as much as possible, i.e., charging B by 20kWh and A by 10kWh in Figure 2b). Thus charging each vehicle by 15kWh is an inefficient allocation. The inability to guarantee an efficient electricity allocation makes neither of these policies efficient market mechanisms for park-and-charge.



(a) inefficient allocation



(b) efficient allocation

Figure 2: The utility difference between two allocations are not considered by pay-per-use and flat-rate pricing policies.

To overcome these shortcomings of existing pricing policies and thus design an efficient market mechanism for the EV park-and-charge mode, we propose to utilize auction as the market mechanism for this mode. An efficient auction can avoid overpricing and underpricing by adapting to the change of demand-supply relation, provide an efficient resource allocation by assigning resources to users who values them most, and thus improve system social welfare. Towards designing an efficient auction mechanism, we need to address the following challenges:

Challenge 1 *The auction framework must be online.* This is because in the EV park-and-charge mode, system information such as the electricity supply, the charging demand of EV users, and the unit-time charging capacity of every vehicle, are stochastic and unpredictable.

Challenge 2 *The auction framework should be computationally efficient.* A good auction should make allocation and pricing decisions in polynomial time so that it can be applied in large-scale facility, e.g., a parking lot with hundreds or thousands charging points.

Challenge 3 *The auction framework must be truthful, individual rational and achieve explicit guarantee on total social welfare.* For an auction, the truthfulness property can avoid speculative strategic action of EV users. The individual rationality ensures that participating users will not receive negative utility. And an explicit guarantees on system social welfare ensures the market efficiency. Directly applying existing auctions such as first-price and VCG auctions into dynamic scenarios of EV charging [13, 14, 24] cannot maintain these properties at the same time, and sometimes even lead to infeasible allocation, e.g., fractional VCG in combinatorial auctions.

To tackle these challenges, we leverage recent progress in mechanism design [5, 6, 16, 23] and propose our *Auc2Charge* online auction framework for park-and-charge in this paper.

3. PROBLEM FORMULATION

We consider an EV park-and-charge station operating in a discrete-time model. Time is divided into time slots of equal length, denoted by $t = 1, 2, \dots, T$. Charging point is equipped at every parking spot. And we consider a set of M EV users, denoted by $j = 1, 2, \dots, M$, who arrive at the station and park their vehicles for one or more time slots. Every EV user j has a budget limit B_j during her whole parking period, During their stay, EV users can submit and update their bids about their charging demand to the station using mobile devices or computers. For each time slot t, any user j can submit up to K bids, denoted by $k = 1, 2, \dots, K$. We define the kth bid submitted by EV user j about charging demand for time slot t as a 2-tuple $(c_j^k(t), b_j^k(t))$, in which $c_j^k(t)$ represents the electricity demand of this bid, and $b_j^k(t)$ represents user j's *reported valuation* for getting charged for an amount of $c_j^k(t)$ electricity in slot t, in a monetary form. Other than the reported valuation, every user j also has a *real valuation* $v_j^k(t)$ for every electricity demand $c_j^k(t)$ submitted to the station. $v_j^k(t)$ is private to user j, which can be affected by many factors, e.g., personal agenda, risk preference and etc., and is also expressed in monetary form. For each user j, her reported valuations $b_j^k(t)$ are correlated across different t, and so are real valuations $v_j^k(t)$. These correlations are private to j. All we know is that $b_j^k(t)$ and $v_j^k(t)$ are monotone increasing functions of $c_j^k(t)$ in any t.

At the beginning of every time slot t, the charging station makes electricity allocation and pricing decisions for slot t based on all the submitted bids of charging demand for t. The total amount of allocated electricity in time slot t must not exceed $R(t)$, the available electricity supply at the station in slot t. For any EV user j, she can win at most one bid among all her submitted bids for every time slot t, and the electricity demand in the winning bids cannot exceed $C_j(t)$, the unit-time maximal charging capacity for EV j during time slot t. This capacity depends on the characteristics of EV battery, e.g., state of charge, lifetime and etc., and hence is stochastic and unpredictable. We use a set of binary decision variables $y_j^k(t) \in \{0, 1\}$ to denote the allocation decision for each bid $(c_j^k(t), b_j^k(t))$. $y_j^k(t) = 1$ means this bid is a winning bid and user j will receive a charging of $c_j^k(t)$ electricity by paying $\Gamma_j(t)$ to the station, and $y_j^k(t) = 0$ means user j does not win this bid and will pay nothing, i.e., $\Gamma_j(t) = 0$. We define $u_j(t)$, the utility for user j at slot t as follows:

$$u_j^t = \sum_{k=1}^{K} v_j^k(t) y_j^k(t) - \Gamma_j(t)$$

To avoid overpricing and underpricing, and to allocate the electricity to EV users who really value it, an good auction mechanism needs to achieve the following properties: 1) truthfulness, 2) individual rationally, and 3) social welfare maximization. An auction mechanism is *truthful* if reporting her *real valuation* as the *reported valuation* for any bid she submits is the dominant strategy for every user j. An auction ensures *individual rationally* if every participating user gets non-negative utility. And an auction achieve social welfare maximization by maximizing the total *real valuation* of all winning bids. When the truthfulness property is achieved, it can be written as $\sum_t \sum_j \sum_k b_j^k(t) y_j^k(t)$ since the reported valuation for each bid equals to the corresponding real valuation [16]. Then we can formulate the offline overall social welfare maximization problem for the park-and-charge system as the following binary integer programming model.

$$\textbf{PNC}: \quad \text{maximize} \quad \sum_{t=1}^{T} \sum_{j=1}^{M} \sum_{k=1}^{K} b_j^k(t) y_j^k(t) \tag{1}$$

subject to

$$\sum_{k=1}^{K} \sum_{t=1}^{T} b_j^k(t) y_j^k(t) \le B_j, \qquad \forall j, \tag{2a}$$

$$\sum_{j=1}^{M} \sum_{k=1}^{K} c_j^k(t) y_j^k(t) \le R(t), \qquad \forall t, \tag{2b}$$

$$\sum_{k=1}^{K} y_j^k(t) \le 1, \qquad \forall j \text{ and } t, \tag{2c}$$

$$\sum_{k=1}^{K} c_j^k(t) y_j^k(t) \le C_j(t), \qquad \forall j \text{ and } t, \tag{2d}$$

$$y_j^k(t) \in \{0, 1\}, \qquad \forall j,\ k \text{ and } t. \tag{2e}$$

In this model, constraint (2a) is the overall budget limit for every user j. Constraint (2b) ensures that the total amount of electricity allocated to the winning bids in time slot t does not exceed the total available electricity supply at the station in t. Constraint (2c) ensures that each EV user j can win at most one bid in every time slot. And constraint (2d) ensures that the electricity allocated to EV user j in time slot t does not exceed j's unit-time charging capacity in t. As we explained earlier, this unit-time maximal charging capacity is unpredictable in that it depends on the characteristics of EV battery, e.g., state of charge, lifetime and etc.

A linear program (LP) relaxation of problem **PNC** can be achieved by replacing the 0-1 integer constraint (2e) with $y_j^k(t) \in [0, 1]$, for any j, k and t. And the upper bound of 1 for each $y_j^k(t)$ can then be omitted because the lower bound 0 and constraint (2c) constitute a sufficient condition of this upper bound. We introduce dual variables x_j, $z(t)$, $s_j(t)$ and $q_j(t)$, for any j and t, to constraints (2a)-(2d) and get the dual problem of the LP relaxation of **PNC** as:

$$\textbf{D-PNC}^{\textbf{lp}}:$$

$$\text{minimize} \sum_{j=1}^{M} B_j x_j + \sum_{t=1}^{T} R(t) z(t) + \sum_{j=1}^{M} \sum_{t=1}^{T} C_j(t) q_j(t) + \sum_{j=1}^{M} \sum_{t=1}^{T} s_j(t), \tag{3}$$

subject to

$$b_j^k(t) x_j + c_j^k(t) z(t) + s_j(t) + c_j^k(t) q_j(t) \ge b_j^k(t), \forall j,\ k \text{ and } t, \tag{4a}$$

$$x_j, z(t), s_j(t), q_j(t) \ge 0, \forall j \text{ and } t. \tag{4b}$$

Solving the **PNC** problem requires the complete knowledge about the whole system over all time slots, which is practically impossible. In the park-and-charge mode, system information including the electricity supply of charging station, the arrival and leaving time of EV users, the bids submitted by EV users and the unit-time charging capacity of different EVs are all stochastic, and thus not known *a priori*. In addition, the overall budget constraint for every EV user makes the bidding decisions at different time slots all coupling together, which further complicates the problem. In addition, the objective function of **PNC** problem in Equation (1) is only valid when the truthfulness property is ensured in the auction mechanism. In the next section, therefore, we propose *Auc2Charge*, an online auction framework for the park-and-charge mode, which ensures the truthfulness and individual rationality and makes electricity allocation and pricing decisions with an explicit competitive ratio on the total system social welfare in polynomial time.

4. AUC2CHARGE: AN ONLINE AUCTION FRAMEWORK

In this section, we present our *Auc2Charge* online auction framework for the park-and-charge mode. This framework utilize the randomized mechanism design technique [5, 6, 16, 23]. Its basic idea is to first transform the offline **PNC** problem into a series of one-shot social welfare maximization problems $\mathbf{PNC_{one}(t)}$, one for each time slot t. During the transformation, the reported valuation $b_j^k(t)$ for the kth bid of EV user j for time slot t is adjusted to a reduced value $w_j^k(t)$ based on the budget limit B_j and the auction result for user j in last time slot $t-1$. In this way, bidding decisions at different time slots are successfully decoupled. For each $\mathbf{PNC_{one}(t)}$ problem, a one-shot randomized auction mechanism, Auc_{one}, is designed to provide a close-form approximation ratio on social welfare of the current time slot, while maintaining the truthfulness and individual rationality. In the following subsections, we first present the transformation from **PNC** to $\mathbf{PNC_{one}(t)}$, which is the basic structure of *Auc2Charge*. We then present the construction of one-shot randomized auction mechanism Auc_{one}. In each subsection, we also analyze the performance of the *Auc2Charge* framework and the Auc_{one} one-shot auction, respectively.

4.1 Basic Structure of Auc2Charge

Assuming all the bids from EV users are truthful bids, as discussed in Section 3, social welfare maximization for EV park-and-charge can be achieved by solving the offline **PNC** problem. To cope with the stochastic and unpredictable nature of system information in **PNC** problem, we propose to transform it into a series of social welfare maximization problem in single time-slot. At a first glance, directly decomposing **PNC** into smaller problems for each time slot seems a nature transformation. However, it is an inappropriate transformation method due to the existence of budget limit across all time slots for every EV user. In direct decomposing, EV users may deplete their budget in early time slots without getting fully charged. Without sufficient budget remaining, EV users will lose the chance to participate the auction in future time slots to get more electricity. This would result in inefficient electricity allocation and thus jeopardizing total social welfare. To avoid this situation, we adopt a *budget-based bid updating* approach, which was first proposed for budget-constraint Adwords auction [6], to perform our transformation. Based on auction results in time slot $t-1$ and the remaining budget of every user, this approach adjusts the reported valuation of each bid in the one-shot social welfare maximization problem in time slot t to a reduced value. In this way, users will not deplete their budget as early as they do in direct decomposing.

Algorithm 1 *Auc2Charge*: the Online Auction Framework for EV Park-and-Charge

1: $\eta_j(0) = 0, \forall j = 1, 2, \ldots, M$
2: **for** $t \to 1, 2, \ldots, T$ **do**
3:   **for all** j, k **do**
4:     **if** $\eta_j(t-1) \geq 1$ **then**
5:       $\omega_j^k(t) = 0$
6:     **else**
7:       $\omega_j^k(t) = b_j^k(t)\Big(1 - \eta_j(t-1)\Big)$
8:     **end if**
9:   **end for**
10:   Run a randomized one-shot auction Auc_{one}. Let $M_{win}(t)$ be the set of winning EVs and k_j be the index of corresponding winning bids for each EV $j \in M_{win}(t)$.
11:   **for** $j \to 1, 2, \ldots, M$ **do**
12:     **if** $j \in M_{win}(t)$ **then**
13:       $\eta_j(t) = \eta_j(t-1)\Big(1 + \frac{b_j^{k_j}(t)}{B_j}\Big) + \frac{b_j^{k_j}(t)}{B_j(\varphi-1)}$
14:     **else**
15:       $\eta_j(t) = \eta_j(t-1)$
16:     **end if**
17:   **end for**
18: **end for**

Using this bid updating approach, we are able to construct the *Auc2Charge* online auction framework as Algorithm 1. In Algorithm 1, we introduce auxiliary variables $\eta_j(t)$ for every EV user j with $\eta_j(0) = 0$. $\eta_j(t)$ is an indicator of the remaining budget of EV user j in time slot t. The more budget is used by user j, the higher value $\eta_j(t)$ becomes. When $\eta_j(t)$ reaches 1, it means user j has used up all her budget B_j. At the beginning of every time slot t, Algorithm 1 adjusts the reported valuation $b_j^k(t)$ to a reduced value $w_j^k(t)$ based on $\eta_j(t-1)$, i.e., Line 3-9. Then it executes a one-shot randomized auction Auc_{one}, which ensures truthfulness, individual rationality and provides an explicit approximation ratio on the one-shot problem $\mathbf{PNC_{one}(t)}$, i.e., Line 10. To not affect the integrity of our discussion on *Auc2Charge*, we leave the design of Auc_{one} auction in next subsection. After getting the result of Auc_{one} in time slot t, *Auc2Charge* accordingly adjusts $\eta_j(t)$, i.e., Line 11-17. If EV user j wins a bid in the auction at time slot t, then $\eta_j(t)$ is updated using the equation in Line 13. In this equation, the parameter φ is defined as $\varphi = (1 + R_{max})^{\frac{1}{R_{max}}}$, where R_{max} is the maximal per-timeslot bid-to-budget ratio, i.e., $R_{max} = \max\{\frac{b_j^k(t)}{B_j}\} \forall j, k$ and t. If user j does not win any bid, $\eta_j(t)$ stays unchanged. Note that the way how φ is defined was first proposed by Buchbinder for revenue maximization in budget-constrained Adwords auction [6]. It is later extended for designing auctions in broader areas [11, 23]. And φ approaches to e when $R_{max} \to 0$.

In Algorithm 1, the one-shot social welfare maximization problem $\mathbf{PNC_{one}(t)}$ for every time slot t is defined as:

$$\mathbf{PNC_{one}(t)}: \quad \text{maximize} \quad p(t) = \sum_{j=1}^{M} \sum_{k=1}^{K} \omega_j^k(t) y_j^k(t), \tag{5}$$

subject to

$$\sum_{j=1}^{M} \sum_{k=1}^{K} c_j^k(t) y_j^k(t) \leq R(t), \tag{6a}$$

$$\sum_{k=1}^{K} y_j^k(t) \leq 1, \qquad \forall j \tag{6b}$$

$$\sum_{k=1}^{K} c_j^k(t) y_j^k(t) \leq C_j(t), \qquad \forall j \tag{6c}$$

$$y_j^k(t) \in \{0, 1\}, \qquad \forall j \text{ and } k. \tag{6d}$$

There are two differences between **PNC** and $\mathbf{PNC_{one}(t)}$. The first one is that the constraint of total budget limit over all time slots, i.e., constraint (2a), is dropped in $\mathbf{PNC_{one}(t)}$. As explained earlier, this constraint is dealt with by the *budget-based bid update* process in Algorithm 1. The second one is that we use $\omega_j^k(t)$, a reduced value of $b_j^k(t)$, in $\mathbf{PNC_{one}(t)}$. Similar as problem **PNC**, we get the linear program (LP) relaxation of problem $\mathbf{PNC_{one}(t)}$ by replacing the 0-1 integer constraint (6d) with $y_j^k(t) \in [0,1]$, for any j and k, and dropping the upper bound 1. Then we define the dual problem of the LP relaxation of $\mathbf{PNC_{one}(t)}$ as:

$$\textbf{D-PNC}^{\mathbf{lp}}_{\mathbf{one}}(\mathbf{t}) : \text{minimize } d(t) = R(t)z(t) + \sum_{j=1}^{M} s_j(t) + \sum_{j=1}^{M} c_j(t)q_j(t) \tag{7}$$

subject to

$$c_j^k(t)z(t) + s_j(t) + c_j^k(t)q_j(t) \geq \omega_j^k(t), \qquad \forall j \text{ and } k, \tag{8a}$$

$$z(t), s_j(t), q_j(t) \geq 0, \qquad \forall j. \tag{8b}$$

$\mathbf{PNC_{one}(t)}$ is NP-hard via a reduction from the 0-1 knapsack problem. However, we can design an α-approximation algorithm for this problem via the greedy primal-dual approach [5], and translate it into a randomized one-shot auction mechanism Auc_{one}, which provides the same approximation ratio for $\mathbf{PNC_{one}(t)}$ and ensures truthfulness and individual rationality [16]. Before discussing how to design Auc_{one}, we propose and prove the following theorem on the performance of the *Auc2Charge* framework.

THEOREM 1. *If we can find a randomized one-shot auction* Auc$_{one}$ *which ensures truthfulness and individual rationality, provides feasible solutions to both* $\mathbf{PNC_{one}(t)}$ *and* $\textbf{D-PNC}^{\mathbf{lp}}_{\mathbf{one}}(\mathbf{t})$*, and guarantees an approximation ratio of* α*, i.e.,* $\alpha p(t) \geq d(t)$*, in every time slot* t*, then the* Auc2Charge *online auction framework provides a* $(1 + R_{max})(\alpha + \frac{1}{\varphi-1})$*-competitive ratio for the* **PNC** *problem.*

PROOF. We follow the sketch in [6,23] to prove this theorem. If we can find such a one-shot auction Auc_{one} satisfying the requirements specified in the theorem, we first prove that the following claims hold.

Claim 1. Let $x_j(t) = \eta_j(t)$ for any t, Algorithm 1 yields a feasible solution to the dual problem $\textbf{D-PNC}^{\mathbf{lp}}$.

It is straightforward to see that Lines 4-8 in Algorithm 1 ensure that $\omega_j^k(t) \geq b_j^k(t)(1 - \eta_j(t-1))$. And the constraint (8a) is guaranteed in the feasible solution provided by Auc_{one}. Therefore, we have

$$c_j^k(t)z(t) + s_j(t) + c_j^k(t)q_j(t) \geq \omega_j^k(t) \geq b_j^k(t)(1 - \eta_j(t-1)),$$

for j, k and t. Because $\eta_j(t)$ is non-decreasing with t, we also have $\eta_j(t) \leq \eta_j(T) = x_j$. Putting it into the right-hand-side (RHS) of the above inequality, we can get

$$c_j^k(t)z(t) + s_j(t) + c_j^k(t)q_j(t) \geq b_j^k(t)(1 - x_j), \forall j, k, \text{ and } t$$

Rearranging its terms and we can see that it is exactly the constraint (4a). Therefore, Algorithm 1 provides a feasible solution to problem $\textbf{D-PNC}^{\mathbf{lp}}$.

Claim 2. Denote $P(t)$ and $D(t)$ as the values of objective functions in **PNC** and $\textbf{D-PNC}^{\mathbf{lp}}$, respectively, after the t-th iteration in Algorithm 1. Let $\Delta P(t) = P(t) - P(t-1)$ and $\Delta D(t) = D(t) - D(t-1)$. Algorithm 1 guarantees that:

$$\frac{\Delta D(t)}{\Delta P(t)} \leq \alpha + \frac{1}{\varphi - 1} \qquad \forall t. \tag{9}$$

To prove this claim, we first see that in the t-th iteration of Algorithm 1, the change in the value of problem $\mathbf{PNC_{ol}}$ is the sum of winning bids in time slot t, i.e., $\Delta P(t) = \sum_{j \in M_{win}(t)} b_j^{k_j}(t)$. Meanwhile, $D(t)$ can be transformed as:

$$\begin{aligned}\Delta D(t) &= \sum_{j=1}^{M_{win}(t)} B_j\Big(x_j(t) - x_j(t-1)\Big) + d(t) \\ &= \sum_{j=1}^{M_{win}(t)} \Big(b_j^{k_j}(t)x_j(t-1) + \frac{b_j^{k_j}(t)}{\varphi - 1}\Big) + d(t) \\ &\leq \sum_{j=1}^{M_{win}(t)} \Big(b_j^{k_j}(t)x_j(t-1) + \frac{b_j^{k_j}(t)}{\varphi - 1}\Big) + \alpha p(t).\end{aligned}$$

Since the only non-zero bids in one-shot auction at t are from EVs that has $x_j(t-1) < 1$, we can rewrite $p(t)$ as

$$p(t) = \sum_{j \in M_{win}(t)} b_j^{k_j}(t)\Big(1 - x_j(t-1)\Big).$$

Putting the rewritten $p(t)$ in the inequality above, we get:

$$\begin{aligned}\Delta D(t) &\leq \sum_{j=1}^{M_{win}(t)} \Big[b_j^{k_j}(t)x_j(t-1) + \frac{b_j^{k_j}(t)}{\varphi - 1} + \alpha b_j^{k_j}(t)\Big(1 - x_j(t-1)\Big)\Big] \\ &= \sum_{j=1}^{M_{win}(t)} \Big(\alpha + \frac{1}{\varphi - 1} - (\alpha - 1)\Big)b_j^{k_j}(t) \\ &\leq \sum_{j=1}^{M_{win}(t)} \Big(\alpha + \frac{1}{\varphi - 1}\Big)b_j^{k_j}(t) \leq \Big(\alpha + \frac{1}{\varphi - 1}\Big)\Delta P(t).\end{aligned}$$

Hence we have proved this claim.

Claim 3. Let $x_j(t) = \eta_j(t)$ for any t. In Algorithm 1, given any EV user j, if time slot t_j^o is the first slot that $\sum_{k=1}^K \sum_{t=1}^{t_j^o} b_j^k(t)y_j^k(t) \geq B_j$, then we have $x_j(t_j^o) > 1$.

We prove this claim by showing that for any EV user j and any time slot $t' = 0, 1, 2, \ldots, T$,

$$x_j(t') \geq \frac{1}{\varphi - 1}\Bigg(\varphi^{\frac{\sum_{k=1}^K \sum_{t=1}^{t'} b_j^k(t)y_j^k(t)}{B_j}} - 1\Bigg). \tag{10}$$

When $t' = 0$, we can easily see that Inequality (10) holds. Assume it holds for $t' - 1$. For time slot t', if $j \notin M_{win}(t)$, i.e., j wins no bid in t', this inequality still holds as both sides stay the same as in $t' - 1$. If $j \in M_{win}(t)$, i.e., j wins one bid in slot t', $x_j(t')$ will be updated in Algorithm 1 as:

$$\begin{aligned}x_j(t') &= x_j(t'-1)\Big(1 + \frac{b_j^{k_j}(t')}{B_j}\Big) + \frac{b_j^{k_j}(t')}{B_j(\varphi - 1)} \\ &\geq \frac{1}{\varphi - 1}\Big(\varphi^{\frac{\sum_{k=1}^K \sum_{t=1}^{t'-1} b_j^k(t)y_j^k(t)}{B_j}} - 1\Big)\Big(1 + \frac{b_j^{k_j}(t')}{B_j}\Big) + \frac{b_j^{k_j}(t')}{B_j(\varphi - 1)} \\ &= \frac{1}{\varphi - 1}\Big(\varphi^{\frac{\sum_{k=1}^K \sum_{t=1}^{t'-1} b_j^k(t)y_j^k(t)}{B_j}}\Big(1 + \frac{b_j^{k_j}(t')}{B_j}\Big) - 1\Big).\end{aligned} \tag{11}$$

Leveraging the inequality $\frac{\ln(1+x)}{x} \geq \frac{\ln(1+y)}{y}$, $\forall 0 \leq x \leq y \leq 1$ and the definition of $R_{max} = \max \frac{b_j^k(t)}{B_j}, \forall j,\ k$ and t, we get

$$1 + \frac{b_j^{k_j}(t')}{B_j} \geq \varphi^{\frac{b_j^{k_j}(t')}{B_j}}.$$

Plugging this conclusion into the RHS of Inequality (11), we have proved Inequality (10) by induction. When $t' = t_j^o$, the RHS of Inequality (10) is greater than or equal to 1 since $\sum_{k=1}^K \sum_{t=1}^{t_j^o} b_j^k(t)y_j^k(t) \geq B_j$, hence we have $x_j(t_j^o) \geq 1$, which completes the proof of this claim.

Claim 4. Algorithm 1 provides an almost feasible solution for the $\mathbf{PNC^{lp}}$ problem.

For any EV j, Algorithm 1 stops it from getting any electricity after t_j^o since its budget has been depleted. And we see that at $t_j^o - 1$, the budget of j is not used up yet. Thus

$$\begin{aligned}\sum_{k=1}^K \sum_{t=1}^T b_j^k(t)y_j^k(t) &= \sum_{k=1}^K \sum_{t=1}^{t_j^o - 1} b_j^k(t)y_j^k(t) + \sum_{k=1}^K b_j^k(t_j^o)y_j^k(t_j^o) \\ &\quad + \sum_{k=1}^K \sum_{t=t_j^o+1}^T b_j^k(t)y_j^k(t)\end{aligned}$$

$$= \sum_{k=1}^{K} \sum_{t=1}^{t_j^o - 1} b_j^k(t) y_j^k(t) + \sum_{k=1}^{K} b_j^k(t_j^o) y_j^k(t_j^o)$$
$$\leq B_j + \max\{b_j^k(t_j^o)\} \leq B_j(1 + R_{max}),$$

for any user j. This indicates that Algorithm 1 ensures a slightly relaxed budget constraint compared to constraint (2a):

$$\sum_{k=1}^{K} \sum_{t=1}^{T} b_j^k(t) y_j^k(t) \leq B_j(1 + R_{max}), \qquad \forall j. \qquad (12)$$

Meanwhile, constraints (2b)(2d)(2e) are strictly guaranteed by Algorithm 1. Therefore, this algorithm yields an almost feasible solution to problem $\mathbf{PNC_{ol}}$.

Claim 4 implies that the total social welfare W_{total} is the minimum between the sum of users' valuation over winning bids and the sum of their budget. Hence we have:

$$W_{total} = \sum_{j=1}^{M} \min\{B_j, \sum_{k=1}^{K} \sum_{t=1}^{T} b_j^k(t) y_j^k(t)\}$$
$$\geq \sum_{j=1}^{M} \min\{\frac{\sum_{k=1}^{K} \sum_{t=1}^{T} b_j^k(t) y_j^k(t)}{1 + R_{max}}, \sum_{k=1}^{K} \sum_{t=1}^{T} b_j^k(t) y_j^k(t)\}$$
$$= \sum_{j=1}^{M} \sum_{k=1}^{K} \sum_{t=1}^{T} \frac{b_j^k(t) y_j^k(t)}{1 + R_{max}} = \frac{P(T)}{1 + R_{max}}.$$

Summing Inequality (9) from Claim 2 over all t and recall that $P(0) = D(0) = 0$, we have $P(T) \geq \frac{D(T)}{\alpha + \frac{1}{\varphi - 1}}$. Plugging it into the RHS of the above inequality, we have

$$W_{total} \geq \frac{D(T)}{(1 + R_{max})(\alpha + \frac{1}{\varphi - 1})} \qquad (13)$$

Therefore by duality, the approximation ratio of *Auc2Charge* is $(1 + R_{max})(\alpha + \frac{1}{\varphi - 1})$. □

4.2 Designing Randomized One-Shot Auction

Having built the basic structure of *Auc2Charge*, we next study how to design a randomized one-shot mechanism Auc_{one} which ensures truthfulness, individual rationality and provides an explicit approximation ratio on the social welfare in t. To this end, we apply the framework proposed in [16] and design the Auc_{one} auction shown in Algorithm 2.

The first step of Auc_{one} is to perform a fractional VCG auction (Line 1-4). This fractional auction follows the classic VCG mechanism to compute the optimal allocation and pricing decisions, and is truthful and individual rational [16,23]. Since its result is inapplicable in real-world, we further decompose this optimal factional solution into a combination of feasible integer solutions to $\mathbf{PNC_{one}(t)}$ (Line 5-6). We utilize the decomposition theory in [7,16] to achieve this. We first define the following linear programming model:

$$\mathbf{DC_P} \text{ minimize } \sum_{l \in I} \lambda_l \qquad (14)$$

subject to

$$\sum_{l \in I} \lambda_l y_j^k(t)^l = \frac{y_j^k(t)^F}{\alpha}, \qquad \forall j \text{ and } k, \qquad (15a)$$

$$\sum_{l \in I} \lambda_l \geq 1, \qquad (15b)$$

$$\lambda_l \geq 0, \qquad \forall l \in I, \qquad (15c)$$

In this problem, every vector $\mathbf{y(t)}^l = \{y_j^k(t)^l\}, \forall j, k$ represents a feasible integer solution to problem $\mathbf{PNC_{one}(t)}$, and α is the approximation ratio provided by the approximation algorithm for $\mathbf{PNC_{one}(t)}$. We notice that there are an exponential number of decision variables in $\mathbf{DC_P}$. Thus we look at its dual problem:

$$\mathbf{DC_D} \text{ maximize } \frac{1}{\alpha} \sum_{j \in M} \sum_{k=1}^{K} y_j^k(t)^F \mu_j^k(t) + \sigma \qquad (16)$$

subject to

$$\sum_{j \in M} \sum_{k=1}^{K} y_j^k(t)^l \mu_j^k(t) + \sigma \leq 1 \qquad \forall l \in I, \qquad (17a)$$

$$\sigma \geq 0, \qquad (17b)$$

and find out that in problem $\mathbf{DC_D}$, there are only $MK + 1$ decision variables but with an exponential number of constraints. Applying the decomposition theory in [7, 16], we have the following lemma:

LEMMA 1. *If there exists an α-approximation algorithm for problem $\mathbf{PNC_{one}(t)}$, a polynomial number of feasible integer solutions and the decomposition $\frac{y_j^k(t)^F}{\alpha} = \sum_{l \in I} \lambda_l \mathbf{y(t)}^l$ can be found within polynomial time by using this approximation algorithm as a separation oracle in the ellipsoid method, and the decomposition satisfies that $\sum_{l \in L} \lambda_l = 1$.*

PROOF. The proof of this lemma follows directly from the decomposition theory in [7, 16], and thus is omitted due to the space constraint. □

Algorithm 2 Auc_{one}: The Randomized One-Shot Auction for EV Park-and-Charge

1: **Step 1: Simulate a fractional VCG auction**
2: Solve problem $\mathbf{PNC_{one}^{lp}}(t)$, the linear relaxation of $\mathbf{PNC_{one}(t)}$ and get the optimal fractional allocation decision $\mathbf{y(t)}^F = y_j^k(t)^F$, for any j and k.
3: For any user j, solve $\mathbf{PNC_{one}^{lp}}(t)$ by setting $w_j^k(t) = 0$ for every k, and denote the optimal value as $\widetilde{V}_j(t)^F$
4: Compute the corresponding payment rule as $\Gamma_j(t)^F = \widetilde{V}_j(t)^F - \sum_{j' \neq j, k} w_{j'}^k(t) y_{j'}^k(t)^F$
5: **Step 2: Decompose the optimal fractional solution**
6: Use the ellipsoid method to solve the primal-dual linear programming problems $\mathbf{DC_P}$ and $\mathbf{DC_D}$, in which an α-approximation algorithm for $\mathbf{PNC_{one}(t)}$ is used as a separation oracle, and get a polynomial number of feasible integer solutions to $\mathbf{PNC_{one}}(t)$. For each solution $\mathbf{y(t)}^l$, get the decomposition coefficient λ_l
7: **Step 3: Construct randomized electricity allocation and pricing decision**
8: Allocation decision: select $y(t)^l$ with probability λ_l
9: Pricing decision: $\Gamma_j(t)^l = \Gamma_j(t)^F \frac{\sum_k w_j^k(t) y_j^k(t)^l}{\sum_k w_j^k(t) y_j^k(t)^F}$

Lemma 1 indicates that in order to get the decomposition of the optimal fractional solution, all we need now is an α-approximation algorithm for problem $\mathbf{PNC_{one}(t)}$. To design such an algorithm, we first drop the constraint (6c) from $\mathbf{PNC_{one}(t)}$ by setting $w_j^k(t) = 0$ for all $c_j^k(t) > C_j(t)$. Correspondingly, dual variables $q_j(t)$ are also dropped from the objective function (7) and constraints (8) of problem $\mathbf{D\text{-}PNC_{one}^{lp}(t)}$. We see that this dropping will not affect the optimal solution to $\mathbf{PNC_{one}(t)}$ or the proof of Theorem 1 since no bids exceeding EV's unit-time maximal charging capacity can win. Combining this dropping process, we resort a classic primal-dual method [5] to design an approximation algorithm for $\mathbf{PNC_{one}(t)}$, as shown in Algorithm 3.

Algorithm 3 A Primal-Dual Approximation Algorithm For $\mathbf{PNC_{one}}(t)$

```
w_j^k(t) = 0 for all c_j^k(t) > C_j(t), drop constraint (6c) and all q_j(t)
y_j^k(t) = 0, s_j(t) = 0, ∀j, k
z(t) = 1/R(t), G(t) = max_{j,k}{c_j^k(t)}, θ = R(t)/G(t)
z_base = e^{θ-1}, M_win = ∅
while R(t)z(t) < z_base and M_win ≠ M do
    for all j ∈ M − M_win do
        k_j = arg max_k {w_j^k(t)}
    end for
    j_win = arg max_j { w_j^{k_j}(t) / (c_j^{k_j}(t) z(t)) }
    y_{j_win}^{k_{j_win}}(t) = 1, s_{j_win}(t) = w_{j_win}^{k_{j_win}}(t)
    r = c_{j_win}^{k_{j_win}}(t) / (R(t) − G(t)), z(t) = z(t) · (z_base)^r, M_win = M_win ∪ {j_win}
end while
```

After the dropping process and the initialization of prime and dual variables, Algorithm 3 adopts a greedy approach to find solutions to $\mathbf{PNC_{one}(t)}$. It iteratively selects the bid with the highest unit-value as a winning bid, one at a time, from EV users who have not yet won any bid in the current time slot. The iterative selection stops when every user has won one bid or the total electricity demand in winning bids exceeds the electricity supply at the park-and-charge station.

Next we analyze the performance of this approximation algorithm. Before deriving its approximation ratio, we first exam the feasibility of solutions computed by this algorithm. Given a $\mathbf{PNC_{one}(t)}$ problem, we assume Algorithm 3 stops at the L-th iteration. Let $s_j^\rho(t)$ and $z^\rho(t)$ be the value of $s_j(t)$ and $z(t)$ in the ρ-th iteration of Algorithm 3, where $\rho = 1, 2, \dots, L$. And we denote the winning EV j_{win} in the ρ-th iteration as j_ρ. We propose the following two lemmas on the feasibility of this algorithm.

LEMMA 2. *After the termination of execution, Algorithm 3 yields a feasible solution to problem* $\mathbf{PNC_{one}(t)}$.

PROOF. The proof of this lemma is straightforward and hence omitted due to the space constraint. □

LEMMA 3. *Define a function*

$$f(z^\rho(t), k_j) \triangleq \frac{w_j^{k_j}(t)}{c_j^{k_j}(t) z^\rho(t)}$$

and $\epsilon = \max_{k',k''=1,\dots,K, j\in M} \frac{c_j^{k'}(t)}{c_j^{k''}(t)}$. *If* $\{s_j^\rho(t), z^\rho(t)\}$ *for any j is a dual solution computed by Algorithm 3 at the end of the ρ-th iteration, then a feasible solution to the problem* $\mathbf{D\text{-}PNC_{one}^{lp}(t)}$ *can be computed as* $\{s_j^\rho(t), \epsilon f(z^\rho(t), k_j) z^\rho(t)\}$ *for any j.*

PROOF. By the end of the ρ-th iteration, it is easy to see that constraint (8a) is satisfied for any EV user $j \in \mathcal{M}_{win}$, since the winning bid of j is the highest bid of all its own bids. For any EV user $j \in M - \mathcal{M}_{win}$, we have

$$f(z^\rho(t), k_{j_\rho}) \geq \frac{w_j^{k_j}}{c_j^{k_j}(t) z^\rho(t)}. \tag{18}$$

From the definition of ϵ, we can find that for any two bids k^1 and k^2, we have $\epsilon c_j^{k^1}(t) \geq c_j^{k^2}(t)$. Thus the constraint (8a) can be satisfied for any EV user $j \in M - \mathcal{M}_{win}$ through substituting $c_j^{k_j}(t)$ by $\epsilon c_j^k(t)$ on the RHS of Inequality (18):

$$f(z^\rho(t), k_{j_\rho}) \geq \frac{w_j^{k_j}}{\epsilon c_j^k(t) z^\rho(t)} \Leftrightarrow c_j^k(t)\epsilon f(z^\rho(t), k_{j_\rho}) z^\rho(t) \geq w_j^{k_j} \geq w_j^k,$$

for any $j \in \mathcal{M}_{win} - M$ and k, which proves this lemma. □

Using these two lemmas on the feasibility of Algorithm 3, we then propose the following theorem on the approximation ratio provided by this algorithm.

THEOREM 2. *For any slot t, Algorithm 3 provides an approximation ratio of α and an integrality gap of α to problem* $\mathbf{PNC_{one}}(t)$ *in polynomial time, where* $\alpha = 1 + \epsilon(e-1)\frac{\theta}{\theta-1}$, *$\epsilon$ is defined in Lemma 3, and θ is defined in Algorithm 3.*

PROOF. It is straightforward to see that Algorithm 3 terminates in polynomial time since the while loop is executed for at most M times and the corresponding loop body can also be finished in polynomial time. Let $p_{opt}(t)$ and $d_{opt}(t)$ be the optimal value of $\mathbf{PNC_{one}}(t)$ and $\mathbf{D\text{-}PNC_{one}^{lp}(t)}$, respectively. We also define $d_1^\rho(t) = \sum_{j=1}^M s_j^\rho(t)$ and $d_2^\rho(t) = R(t) z^\rho(t)$. We follows the sketch in [5] to prove the approximation ratio of α by considering three different cases..

Case 1: When Algorithm 3 stops at the L-th iteration, $\mathcal{M}_{win} = M$ and $R(t)z(t) < z_{base}$. In this case, every EV wins one bid. We also see that for each EV user j, only its highest bid $w_j^{k_j}(t)$ will be considered during the allocation process. Thus, we observe that 1) $p(t) = \sum_{j\in M} w_j^{k_j}(t)$, and 2) $s_j(t) = w_j^{k_j}(t) \geq w_j^k(t)$ for any EV j and its bid k. The second observation makes any non-negative $z(t)$ become a part of feasible solution to problem $\mathbf{D\text{-}PNC_{one}^{lp}(t)}$. By weak duality, we have $d(t) \geq p(t)$. When we let $z(t) = 0$, $d(t)$ reaches its optimal value $d_{opt}(t)$ and has the exactly the same value as $p(t)$, i.e., $\sum_{j\in M} w_j^{k_j}(t)$. In this way, Algorithm 3 provides an optimal solution to problem $\mathbf{PNC_{one}}(t)$.

Case 2: When Algorithm 3 stops at the L-th iteration, $R(t)z(t) \geq z_{base}$, and there exists an iteration $\rho \leq L$ such that $\alpha \geq \frac{d_{opt}(t)}{d_1^{\rho-1}(t)}$. In this case, we have $d_1^{\rho-1}(t) = p^{\rho-1}(t)$. And we also observe that $d_1^\rho(t)$ is non-decreasing as ρ increases. By weak duality, Algorithm 3 provides an approximation ratio of α in this case.

Case 3: When Algorithm 3 stops at the L-th iteration, $R(t)z(t) \geq z_{base}$, and for any iteration $\rho \leq L$, $\alpha < \frac{d_{opt}(t)}{d_1^{\rho-1}(t)}$. In this case, we first define two auxiliary variables:

$$\delta = \left(\frac{R(t)}{G(t)} - 1\right)\left(z_{base}^{1/(\frac{R(t)}{G(t)}-1)} - 1\right) \text{ and } \Delta = \frac{\delta R(t)}{R(t) - G(t)}.$$

Leveraging the update process of $z^\rho(t)$ in Algorithm 3 and the inequality $(1+a)^x \leq 1 + ax, \forall x \in [0.1]$, we have the following observation on $d_2^\rho(t)$:

$$\begin{aligned} d_2^\rho(t) &= R(t) z^{\rho-1}(t) z_{base}^{c_{j_\rho}^{k_{j_\rho}}(t)/(R(t)-G(t))} \\ &= R(t) z^{\rho-1}(t) \left(1 + \frac{\delta}{\frac{R(t)}{G(t)} - 1}\right)^{c_{j_\rho}^{k_{j_\rho}}(t)/G(t)} \\ &\leq R(t) z^{\rho-1}(t)\left(1 + \frac{\delta}{\frac{R(t)}{G(t)}-1} \cdot \frac{c_{j_\rho}^{k_{j_\rho}}(t)}{G(t)}\right) \\ &= d_2^{\rho-1}(t) + \Delta c_{j_\rho}^{k_{j_\rho}}(t) z^{\rho-1}(t). \end{aligned} \tag{19}$$

From the execution of Algorithm 3, we also observe that

$$p^\rho(t) = p^{\rho-1}(t) + \omega_{j_\rho}^{k_{j_\rho}}(t), \text{for any } 0 < \rho \leq L.$$

Using the definition of function f and Inequality (18), we continue to transform the RHS of Inequality (19):

$$d_2^\rho(t) \leq d_2^{\rho-1}(t) + \frac{\Delta(p^\rho(t) - p^{\rho-1}(t))}{f(z^{\rho-1}(t), k_{j_\rho})}. \tag{20}$$

Lemma 3 shows that any set of dual solution $\{s_j^{\rho-1}(t), z^{\rho-1}(t)\}$, $\forall j$ during the execution of Algorithm 3 can be transformed to a feasible dual solution to $\mathbf{D\text{-}PNC_{one}^{pl}(t)}$. Thus we have

$$d_{opt}(t) \leq d_1^{\rho-1}(t) + \epsilon f(z^{\rho-1}(t), k_{j_\rho}) d_2^{\rho-1}(t),$$

which implies

$$f(z^{\rho-1}(t), k_{j_\rho}) \geq \frac{d_{opt}(t) - d_1^{\rho-1}(t)}{\epsilon d_2^{\rho-1}(t)}.$$

Since in this case we have $\alpha < \frac{d_{opt}(t)}{d_1^{\rho-1}(t)}$, we can further get

$$f(z^{\rho-1}(t), k_{j_\rho}) \geq \frac{(\alpha-1)d}{\epsilon \cdot \alpha d_2^{\rho-1}(t)}. \tag{21}$$

Plugging Inequality (21) into the RHS of Inequality (20) and utilizing $1 + x \leq e^x, \forall x \geq 0$, we get the following inequality

$$\begin{aligned} d_2^\rho(t) &\leq d_2^{\rho-1}(t)\left(1 + \frac{\epsilon\cdot\alpha\cdot\Delta}{(\alpha-1)d_{opt}(t)}(p^\rho(t) - p^{\rho-1}(t))\right) \\ &\leq d_2^{\rho-1}(t) e^{\frac{\epsilon\cdot\alpha\cdot\Delta}{(\alpha-1)d_{opt}(t)}(p^\rho(t)-p^{\rho-1}(t))}. \end{aligned} \tag{22}$$

Summing Inequality (22) over $\rho = 1, 2, \dots, L$, we reach

$$d_2^L(t) \leq d_2^0(t) e^{\frac{\epsilon\cdot\alpha\cdot\Delta}{(\alpha-1)d_{opt}(t)} p^L(t)}.$$

Note that $d_2^0(t) = 1$, and when the algorithm stops at the L-th iteration under this case, we have $d_2^L(t) = R(t)z(t) \geq z_{base}$. Hence we have

$$\theta - 1 = \frac{R(t)}{G(t)} - 1 \leq \frac{\epsilon\cdot\alpha\cdot\Delta}{(\alpha-1)d_{opt}(t)} p^L(t) \Leftrightarrow \frac{d_{opt}(t)}{p^L(t)} \leq \frac{\epsilon\cdot\alpha\cdot\Delta}{(\alpha-1)(\theta-1)}.$$

Based on weak duality we know that

$$\frac{p_{opt}(t)}{p^L(t)} \le \frac{d_{opt}(t)}{p^L(t)},$$

which means $\frac{d_{opt}(t)}{p^L(t)}$ is an upper bound of the approximation ratio. Through some simple mathematical transformation, we can obtain the following approximation ratio

$$\alpha = 1 + \epsilon(e-1)\frac{\theta}{\theta - 1}.$$

Denote the optimal value of the linear relaxation version of $\mathbf{PNC_{one}}(t)$ problem as $p_{opt}^{LR}(t)$. From the fact that $d_{opt}(t) \ge p_{opt}^{LR}(t)$ and $p_{opt}(t) \ge p^L(t)$ we can further get

$$\frac{p_{opt}^{LR}(t)}{p_{opt}(t)} \le \frac{p_{opt}^{LR}(t)}{p^L(t)} \le \frac{d_{opt}(t)}{p^L(t)} = \alpha$$

Hence, the integrality gap provided by Algorithm 3 is also α and we have finished our proof. □

Having proved the approximation ratio and the integrality gap provided by Algorithm 3, we can now plug it into the Auc_{one} one-shot auction as a separation oracle. In this way, Auc_{one} can derive a polynomial number of feasible integer solutions to the $\mathbf{PNC_{one}(t)}$ problem and the corresponding decomposition coefficient through ellipsoid method, as shown in Lemma 1. If the Auc_{one} mechanism is performed independently, i.e., in a scenario without any budget-based bid updating, it can achieve an α-approximation ratio on social welfare. This conclusion was proved in [16]. With the proposed budget-based bid updating process, however, the approximation ratio provided by Auc_{one} will be scaled up by a factor of $1 + R_{max}$. This is because the valuation $w_j^k(t)$ in problem $\mathbf{PNC_{one}(t)}$ is a reduced value from $b_j^k(t)$. Therefore, we propose the following theorem on the performance of Auc_{one} in our online auction framework.

THEOREM 3. Auc_{one} *is computationally efficient, truthful, individual rational, and* $\alpha(1 + R_{max})$*-competitive.*

PROOF. The proof of this theorem follows the sketch in [16, 23], and is omitted due to the space constraint. □

Putting the conclusion in Theorem 1 and 3 together, we then propose the following theorem on the performance of *Auc2Charge* online auction framework.

THEOREM 4. *Integrating* Auc_{one} *into Algorithm 1, our* Auc2Charge *framework is truthful, individual rational, computationally efficient and* $(1 + R_{max})(\alpha(1 + R_{max}) + \frac{1}{\varphi - 1})$*-competitive.*

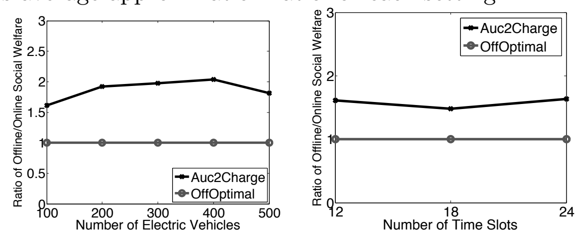
PROOF. The proof of this theorem follows the sketch in [6, 23] and is omitted due to the space constraint. □

5. NUMERICAL SIMULATION

We conduct numerical simulation to demonstrate the efficacy of the *Auc2Charge* online auction framework. In our simulation, we define the length of one time slot as one-hour. We assume a parking-lot with 500 spots, each of which has a charging point, and the electricity source of this facility is from renewable energy. We use the hourly wind power generation capacity profile of a 24-hour period from a random location in New York City of the United Sates [1] as the electricity supply profile of this facility. We assume each EV has the same battery capacity of $40kWh$. In any simulation with T time slots, we assume all EVs arrive at the charging stations between time slot 1 and $T - 6$ in a uniformly random way. The SOC of each EV when arriving at the charging facility is uniform randomly chosen between 0 and 0.7. And the length of parking for each EV is uniformly chosen between 2 to 6 hours. The total budget of each EV user follows a uniform distribution between 8 and 12 dollars. At the beginning of each time slot, every EV user submits up to 5 bids in the form of *(valuation, charging demand)*. For every EV user j, the valuation and charging demand in her bids for a given time slot t are separately randomly generated. Then the valuations and charging demands are sorted respectively and reorganized based on the sorted order to ensure the monotonicity of valuation over charging demand. The unit-time maximal charging capacity for each EV at every time slot is set to be a random value between $6kWh$ and $8kWh$. We perform simulation of *Auc2Charge* under the setting of $T = 12, 18$, and 24 hours, with 5 different numbers of EVs, i.e., 100, 200, 300, 400 and 500. simulation for 5 times and compute the average value.

5.1 Social Welfare

We first investigate the performance of *Auc2Charge* on social welfare. To this end, we first simulate *OffOptimal*, the offline optimal solution to the **PNC** problem. We then repeat the simulation of *Auc2Charge* framework under each combination of T and EV numbers for 5 times and derive its average approximation ratio for each setting.



(a) $T = 12$ (b) 100 Electric Vehicles

Figure 3: Offline/Online Ratio of Social Welfare

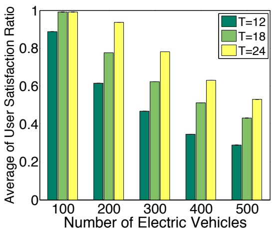
Figure 3a) shows the approximation ratio of social welfare achieved by *Auc2Charge* under different numbers of participating EVs when the simulation period is 12 hours. We see that our *Auc2Charge* online auction achieves a stable offline/online ratio on social welfare under different density of electric vehicles. In Figure 3b) we plot the approximation ratio of *Auc2Charge* under different simulation periods when the number of EV is fixed at 100 . And we can observe a similar stable approximation ratio provided by *Auc2Charge*. Not only does these two observations indicate the existence of approximation ratio provided by our *Auc2Charge* auction, they also demonstrate the scalability of this online auction under various numbers of participating vehicles and arbitrary time period. It thus sheds some light for large-scale deployment of *Auc2Charge* in practice.

5.2 User Satisfaction

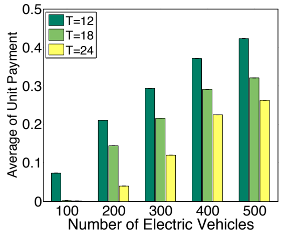
Next we study the performance of our *Auc2Charge* auction framework from the perspective of EV users. Note that we do not include results of *OffOptimal* because it is impractical in real world. We focus on the following metrics regarding the experience of EV users during the parking:

- *User Satisfaction Ratio*: The ratio between the total electricity allocated to an EV and the amount of electricity needed to fully charge this EV.
- *Unit Charging Payment*: The average payment made by an EV to charge one unit of electricity, i.e., 1kWh.
- *Total Charging Payment*: The total charging payment made by an EV during its parking.
- *Budget Utilization Ratio*: The ratio between the total charging payment of EV and its total bidding budget.

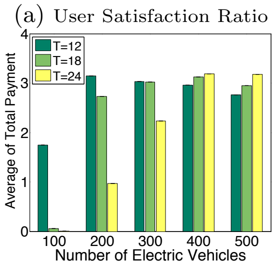
In what follows, we discuss the performance of *Auc2Charge* on these metrics. Figure 4a) shows the average user satisfaction ratio when executing the *Auc2Charge* framework. When the number of EVs are fixed, the average user satisfaction ratio increases as the number of time slots increases. Because the arrival of EVs are distributed between time slot 1 and $T - 6$ in our simulation. When T becomes larger, there are less EVs staying in the parking lot in a given time slot, resulting in abundant supply of electricity and a high user satisfaction ratio. On the contrary, for a fixed length of simulation time, this ratio decreases as the number of EVs increases. This is because with more vehicles arriving at the parking lot, the electricity becomes scarce. We then plot the unit charging payment of EV users under different simulation settings in Figure 4b). We can see that the average unit charging payment of EV users shows an opposite monotonicity from user satisfaction ratio, i.e., the unit charging payment increases as the number of EVs increases, and decreases as the simulation period T decreases. This monotonicity is desirable in an efficient market mechanism as it plays the role of *the indicator of electricity scarcity*. As a conclusion, both the monotonicity on user satisfaction ratio and unit charging payment demonstrate the ability of *Auc2Charge* to adapt to various demand-supply relations.



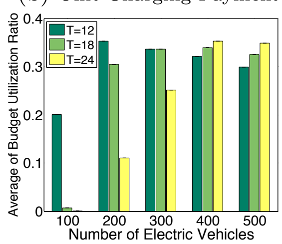
(a) User Satisfaction Ratio



(b) Unit Charging Payment



(c) Total Charging Payment



(d) Budget Utilization Ratio

Figure 4: Metrics of User Satisfaction

Figure 4c)-d) show the average total payment and the average budget utilization ratio of EV users in different simulation settings. Some interesting observations in these two figures are worth noting. The first one is that the total payment and budget utilization ratio does not show any monotonicity on simulation period or the number of EVs. For instance, the highest budget utilization ratio occurs when 200 EVs participate in the auction in a 12-hour period. This is because the expense of EV users is decided by both the total amount of electricity they received and the unit-price they pay. As we explained, these two factors have the opposite monotonicity, which leads to the lack of regular pattern in total payment and budget utilization ratio. The second observation is that when there are only 100 EVs in the simulation and the simulation time is large, i.e., 18 or 24 hours, the total payment of EV users decided by *Auc2Charge* online auction approaches to zero, and so is the budget utilization ratio. This is because under these two cases, the small number of EVs and the long simulation period cause the oversupply and under-demand of electricity, resulting in a less competition between EV users to get charged. Under this scenario, *Auc2Charge* is able to allocate electricity to fulfill all the charging demand of EV users, i.e., a 100% user satisfaction ration in Figure 4a), while charging a near-zero unit price to EV users as shown in Figure 4b). The near-zero total charging payment and budget utilization ratio are direct response from *Auc2Charge* on electricity oversupply and under-demand, and protects EV users from overpricing. When the electricity demand is much larger than the supply, on the contrary, *Auc2Charge* allocates limited electricity to EV users who really value the electricity with a higher pricing, which protects the charging station from underpricing. One example can be found in Figure 4b)-d) when $T = 12$ and the number of EVs is 500. Observations on user satisfaction metrics in these scenarios again demonstrate *Auc2Charge*'s ability to adapt to various supply-demand relations. And as we already showed in Figure 3, *Auc2Charge* achieves this adaptiveness with a social welfare guarantee simultaneously.

6. RELATED WORK

EV Charging Facilities. There has been a growing literature on the operation mode of EV charging facilities [2–4, 8, 9, 12, 17, 19]. Lopes *et al.* [17] explored the potential benefits and impact brought by the integration of EV into power grid. Chen *et al.* [9] designed a central controller to schedule the EV charging using renewable energy. The authors proposed an online scheduling algorithm that can achieve the maximum competitive ratio of an offline solution. Ardakanian *et al.* [4] designed a distributed charging algorithm to adjust EV charging rate based on available capacity of power network and ensure the proportional fairness between EV chargers. Gan *et al.* [12] proposed a distributed controller to capture the uncertainty and elasticity of EV charging and the intermittency of renewable energy. Chen *et al.* [8] studied a joint optimal power flow and EV charging problem. An online distributed controller was designed to enable efficient EV charging and maintain grid stability.

Recently, a promising operation mode called park-and-charge was proposed, and has drawn the interest of both academia and industry. It allows EVs to get charged during staying in a parking lot while the drivers are away for other agendas. Several experiments have been performed to explore the potential of this mode in integrating EVs into smart grid and ITS [2, 3, 19]. The V-charge project [3] proposed to design an automated valet park-and-charge system to support local autonomous transportation. And some prototypes have been built in several universities in Europe. GM and TimberRock built an park-and-charge station at GM's E-Motor Plant [2], where TimberRock manages the charging schedule of a fleet of EVs with the aim to balance the intermittent renewable energy supply and the stochastic EV charging demand. The U.S. Air Force is conducting an experiment at its Los Angeles Base [19], where the charging schedule of an EV fleet is controlled to minimize the total electricity cost. Though these experiments provide positive outcome, the missing of an efficient market mechanism prevents it from large-scale deployment. Existing pricing policies such as pay-per-use and flat-rate fail to adapt to the dynamic change of demand-supply relation in this mode. To fill this gap, we propose the *Auc2Charge* online auction framework as the market mechanism for park-and-charge

Auction Theory. Being a market mechanism, auction allocates resources to buyers who value them most, reduces the chance of overpricing and underpricing, and thus improves social welfare. Auction mechanisms have been widely used in areas such as online advertisement [6, 11], wholesale electricity market [20, 22] and cloud computing [23, 25, 26]. Recently some studies [13, 14, 21, 24] focus on utilizing auction mechanisms into different scenarios of EV charging, with the hope of improving the efficiency of electricity allocation. For instance, Gerding *et al.* [14] proposed a two-side market with advanced reservation, in which EV users and the charging station can exchange their charging preference and cost. An online mechanism was designed for this market to ensure the truthfulness of EV users, but it does not provide any explicit guarantee on system social welfare. Robu *et al.* [21] designed an online mechanism, in which EV users bid for different charging speeds based on their arrival time, and cancel the charging allocation on departure. The authors analyzed the worst-case competitive ratio of social welfare under such cancellation scenario. Designing an auction with a close-form approximation ratio on social welfare while ensuring truthfulness and individual rationality has always been a major challenge in mechanism design. Lavi *et al.* [16] tackles this challenge by proposing a randomized auction framework, which could translate any α-approximation algorithm into truthful and individual rational mechanisms. In our paper, we leverage this framework together with techniques in Adwords auction [11, 23] and combinatorial auction [5], and propose the *Auc2Charge* online auction framework for electricity allocation in EV park-and-charge station. We show that *Auc2Charge* ensures truthfulness and individual rationality, and provides a close-form approximation ratio on total social welfare in polynomial time. *Auc2Charge* was motivated by the online auction mechanism for resource allocation in cloud computing [23]. One important difference is that in cloud systems, users do not have any constraint on the capacity of receiving resources. In EV charging systems, however, there exists a stochastic and unpredictable constraint on the unit-time maximal charging capacity for every EV due to battery characteristics. To the best of our knowledge, *Auc2Charge* is the first online auction mechanism that achieves truthfulness, individual rationality and explicit guarantee on social welfare for electricity allocation in EV park-and-charge.

7. CONCLUSION

As a promising operation mode for charging stations, park-and-charge allows EVs to get charged during their stay in a parking lot. To find an efficient market mechanism for large-scale deployment of this mode, we explore the feasibility and benefits of utilizing auction in EV park-and-charge. In an auction, every EV user can submit and update bids about their charging demand to the parking lot, which makes electricity allocation and pricing decisions based on the collected bids. We propose *Auc2Charge*, an online auction framework for park-and-charge. Our theoretical analysis indicates that *Auc2Charge* ensures truthfulness and individual rationality, computes electricity allocation and pricing solutions in a polynomial time, and guarantees an explicit approximation ratio of system social welfare. Results from numerical simulation demonstrate the efficacy of *Auc2Charge* in terms of social welfare and user satisfaction. The *Auc2Charge* auction framework fills the gap between small-scale experiment and large-scale real-world deployment of park-and-charge mode. Though it is designed for park-and-charge, the design rationale of *Auc2Charge* also applies to other modes for charging stations, e.g., the charging-point reservation system. As future work, we plan to extend the *Auc2Charge* framework by including other realistic constraints in both the electricity market, e.g., vehicle-to-grid electricity transmission and ramp-up/ramp-down cost of electricity generation, and intelligent transportation systems, e.g., the uncertainty of EV's mobility.

8. ACKNOWLEDGMENT

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