On-Line Event-Driven Scheduling for Electric Vehicle Charging via Park-and-Charge

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***Abstract*—Large-scale charging stations become indispensable infrastructure to support the rapid proliferation of electric vehicles. Their operation modes have drawn great attention from both academia and industry. One promising mode called park-and-charge has been recently introduced. This new mode allows customers to park their electric vehicles at a parking lot, where the vehicles are charged during the parking time. Several small-scale experiments, such as the V-Charge project and General Motors' E-Motor plant, have demonstrated its potential. A key enabler for deploying this mode to large-scale stations is effective and efficient charging load scheduling methods. Most existing works confine to the time-driven scheduling policy due to their sole focus on the charging service. Applying their solutions to the park-and-charge mode would jeopardize the unitization of charging resource or cause frequent charging mode switching. This inapplicability motivates us to explore the feasibility and benefits of exploiting the event-driven scheduling policy in park-and-charge systems. Further, to better characterize charging load in this mode, we propose to adopt a metered model, by which a system gains value in proportion to the served charging demand. To be specific, the objective of this paper is to carry out both theoretical and experimental analysis for event-driven algorithms adapted to this metered model. We leverage both the competitive analysis and resource augmentation to demonstrate the non-constant and constant performance bounds for the earliest-deadline-first and highest-value-first algorithms respectively. Moreover, we provide a stronger theoretical result, i.e., the performance bound for the whole class of work-conserving scheduling algorithms. Through extensive simulations, we validate the proposed theoretical results and further provide interesting findings from the in-depth analysis of the simulation results.**

I. Introduction

The market share of electric vehicles (EVs) keeps proliferating due to the benefits of few emissions and high power efficiency. As forecasted by research reports such as [1], [2], the growth of electric vehicles in the globe would reach 7 million per year by 2020, and electric vehicles would hold 28% of the U.S. vehicle market by 2031. The associated high impact on the power grid and transportation system has drawn great interests from both the industry and academia. Various charging facilities have been studied, such as charging points in residential areas and working places [3]–[9]. Among them, charging stations have become indispensable infrastructure to support the deep integration of electric vehicles [7]–[11]. Thus, their operation mode needs careful design.

Due to the relatively long and frequent charging cycles, one promising operation mode, named park-and-charge, has been recently proposed for electric vehicle charging [7]–[9], [12], [13]. By this mode, parking lots are equipped with charging points to function with both parking and charging services. Electric vehicles can receive charging during the period of parking. One key advantage of this mode is that customers can directly follow their agendas after parking their vehicles with no need to care about the charging process. Potential applications of this park-and-charge mode include parking lots at office buildings, shopping malls, and airports. Recently, several field experiments have been conducted to explore the feasibility of the park-and-charge mode. For example, several universities in Europe together with The Bosch Group carry out the project V-Charge, which aims to develop an automated valet parking and charging system to support autonomous local transportation [12]. General Motors (GM), OnStar and TimberRock collaborate to perform a pioneering experiment at GM's E-Motor Plant, whose objective is to coordinate the charging demand of parked electric vehicles with the co-located renewable generation [13]. These small-scale experiments present positive feedbacks on the potential of the park-and-charge mode. An important step to deploy this mode to large-scale charging stations is to develop effective and efficient charging load scheduling methods.

Although researchers have begun to take efforts towards this step, most existing works solely focus on the charging service and thus confine to the time-driven scheduling policy, e.g., [14]–[19]. By this policy, the time line is equally slotted and scheduling decisions are made for each time slot. A key assumption in these works is that the arrival time and deadline (i.e., the time when the user picks up her/his EV) of an EV are right at the beginning or the end of a time slot. Under this assumption, one major dilemma for applying this policy to park-and-charge systems is to determine the length of time slots. Long time slots lead to few charging mode switchings but cause under-utilized charging points at the stations, while short time slots improve charging point utilization but cause many mode transitions for electric vehicles. Frequent mode switchings can significantly compromise the lifetime and capacity of EV batteries. Hence, to avoid such dilemma, we propose to explore the event-driven scheduling policy to schedule electric vehicle charging in park-and-charge systems in this paper.

Event-driven scheduling is widely recognized in real-time community, and thus many real-time scheduling methods such as earliest-deadline-first (EDF) and least-laxity-first (LLF) have been designed along with this policy. Further, these scheduling methods have been extensively studied in various kinds of systems such as embedded systems, communication systems [20], and even server farms/data centers [21]. A clear vision is, however, still elusive regarding the feasibility and benefits of applying real-time scheduling to park-and-charge systems. Understanding and analyzing such application is a non-trivial task because of three key challenges. First, modeling charging tasks should capture both the parking and charging functions. Second, charging tasks usually have different temporal constraints and may also have different values (e.g., utilities to measure user satisfaction) due to different energy prices paid by the customers. Choosing between temporality-based and value-based scheduling methods is involved. Third, the scheduling methods should be online, which need to

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determine vehicles for charging based only on the information of electric vehicles that have already arrived at the system.

To address these challenges, we first propose to adopt a metered model, by which the system gains in proportion to the amount of already served demand of a charging task. This metered model better fits to park-and-charge, compared with the classical deterministic model (detailed model discussion in Section II-B). Second, we study the adaption of several widely used on-line scheduling methods to the metered model. They include: (i) temporality-based algorithms such as EDF and shortest-job-first (SJF); (ii) a value-based algorithm, i.e., highest-utility-first (HUF). Their pros and cons regarding park-and-charge are thoroughly discussed using: (i) theoretical analysis via the competitive analysis and resource augmentation techniques; (ii) experimental validations via detailed simulations. To be specific, our major contributions are as follows.

• We novelly adapt several well-known on-line scheduling algorithms to schedule EV charging in park-and-charge systems. As to maximizing EV user satisfaction, we demonstrate (i) the non-constant performance bound of EDF and the constant bound of HUF; (ii) a generalized theoretical result, i.e., the performance bound for the whole class of work-conserving scheduling algorithms.

• We conduct extensive simulations to make comparison on optimizing EV user satisfaction. The results show the near-optimality and performance consistency of the value-based algorithm (i.e., HUF), and the sub-optimality and performance anomaly of temporality-based algorithms (e.g., EDF and SJF).

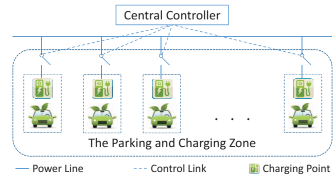
Note that this paper targets the case of multiple charging points, which is equivalent to the multiple processor case as to the scheduling theory. From this perspective, our work is an extension to [22]. Work [22] carries out competitive analysis for HUF solely, while our work performs both competitive analysis and resource augmentation analysis for several well-known algorithms (including HUF) and further for the whole class of work-conserving algorithms.

The rest of the paper is organized as follows. Section II presents the system model and problem statement. Sections III and IV analyze algorithms using competitive analysis and resource augmentation respectively. Section V evaluates the studied on-line algorithms. Section VI concludes the paper.

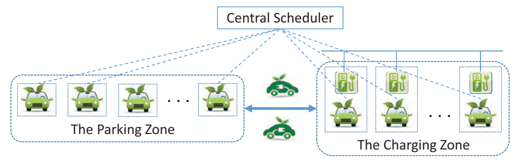
II. Problem Description

There are usually two different implementations for a park-and-charge system. The first one, a combined zone (Fig. 1(a)), is that the parking zone and charging zone are in the same area, and each parking space is equipped with one charging points [4], [5]. A centralized controller controls power on and off for each charging point. The second one, separate zones (Fig. 1(b)), is that the parking zone and charging zone are at different areas [12], [23]. The parking zone has no charging infrastructure. Charging points only concentrate in the charging zone and always have power supply. A centralized scheduler decides which electric vehicles to charge at the charging zone and which to park at the parking zone. This implementation needs valets to assist moving vehicles or allows fully autonomous driving between the two separate zones [12].

For the customers, they check in and drop off their electric vehicles at the parking zone. Then, they can leave and directly proceed to follow their original agendas, instead of taking care



(a) A combined zone



(b) Separate zones

Fig. 1. The schematics of two different implementations of park-and-charge.

of charging their vehicles. During their absence, the park-and-charge system is responsible to schedule vehicles for charging. Upon the returns, they pick up their vehicles from the parking zone, where the vehicles are already waiting for them. For the system operator, the checked-in vehicles need to be scheduled according to users' charging requirements. This work focuses on the charging scheduling problem: to determine when to charge which electric vehicles. The scheduling decisions can be realized by (i) switching power on or off for the corresponding charging points as to the combined-zone case in Fig 1(a), and (ii) moving the corresponding vehicles into or out of the charging zone as to the separate-zone case in Fig. 1(b).

In this section, we first present a general system model that applies to both of the cases. Then, we use model differentiation to show that the model better fits the park-and-charge scenario, and lastly provide problem statement.

A. System Modeling

We consider a park-and-charge system that supports at most M charging points turned on currently. This limit derives from the physical constraints such as respecting the capacity of upstream transformers [6], or is imposed by the system operators such as capping the peak power of the systems [3], [24]. Without loss of generality, we assume that M is less than the number of parking spots [4], [9], [23]. Each charging point supports a charging power of p. Each electric vehicle is modeled by a charging task EV_i, which is characterized by a four-tuple (a_i, d_i, e_i, u_i). The following first details the definition of each parameter and then provides the accommodation of this model to realistic charging behavior of EV battery.

Parameter Definition. (i) The arrival time a_i is when the customer drops off his/her vehicle at the parking zone. At this time, the vehicle is also ready for charging.

(ii) The charging demand e_i is the amount of energy that the user expects after the deadline. It can be estimated by $(soc_i' - soc_i) \cdot B_i$, where soc_i is the state of charge (SOC) at the arrival time a_i; soc_i' is the expected SOC after deadline d_i; B_i is the capacity of the vehicle's battery. SOC is usually denoted as a percentage value. Existing methods such as [25], [26] can be used to estimate SOC of EV battery.

(iii) The deadline d_i is a user-specified time, after which he/she will come back to pick up his/her vehicle. The deadline is soft, that is, after it, the system stops charging the vehicle and switch off the power supply or park it at the parking zone. If with partially completed demand, the system gains the partial value. Further, it uses arbitrary deadline. That is, there is

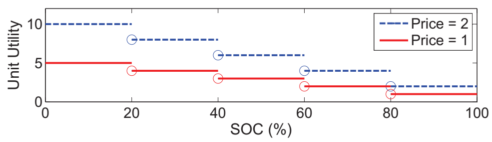


Fig. 2. An example piecewise linear function for the unit utility, which increases as to the energy price and decreases as to the SOC.

no constraint between the deadline d_i and charging time e_i/p, i.e., the former can be larger than, less than, or equal to the latter. This setting can largely avoid cheating as to customers specifying their charging requirements, e.g., deadline. Since a vehicle receives no energy after the deadline, the vehicle may receive little energy in the end if the customer deliberately reports a much shorter deadline than e_i/p.

(iv) The parameter u_i denotes unit utility. If an EV receives an amount x of energy, the gained utility is $u_i \cdot x$. The utility is a measurement of user satisfaction or the value of a charging task, which depends on both the energy price and state-of-charge. First, to differentiate users' charging requirements, the system may employ some market mechanisms such as pricing or biding to determine energy price for individual charging tasks[1] [27], [28]. Thus tasks may have different energy prices. Customers willing to pay more usually come with higher value about the charging service [27], [28]. Second, user satisfaction is usually assumed to follow the law of diminishing utility [19], [29], where the marginal utility decreases as state-of-charge increases. Hence, the unit utility can be modeled as a function that is non-decreasing as to energy price and non-increasing as to state-of-charge. This kind of modeling approach has been widely used in EV charging studies [19], [29]–[31] and network economics literature [32]–[34].

Model Accommodation for Realistic Non-linear Charging Behavior. In reality, SOC of EV battery is non-linear with time. For accommodating to this, we can split a charging task into a sequence of subtasks that arrive sequentially at the system. We use an approximation: for each subtask, SOC or the received energy of an EV is linear with time. Further, the unit utility function can be approximated by a piecewise-constant function that respects the above non-decreasing and non-increasing features. Each subtask corresponds to a piece and thus has a constant unit utility. Fig. 2 shows a numerical example for this accommodation. By this example, for a subtask with SOC in $[0, 20\%]$, the unit utility is 10 if energy price is 2 (the dotted lines) and it is 5 if price is 1 (the solid lines); for that in $(20\%, 40\%]$, it is 8 if price is 2 and 4 if price is 1; and so on. The gained utility of a charging task EV_i is thus equal to the sum of each subtask's utility, i.e., $\sum_j u_{ij} \cdot x_{ij}$, where u_{ij} and x_{ij} are the unit utility and received energy of subtask EV_{ij} respectively. Note that the following results remain true with this accommodation. For ease of presentation, we associate each electric vehicle with one charging task and omit the subtask index in the following algorithm analysis.

The system allows preemption of the charging process of electric vehicles, and each preemption corresponds to one charging mode switching or mode transition. That is, the system is able to suspend an electric vehicle's charging process by turning power off for its charging point as to the combined-zone case or re-parking the vehicle at the parking zone as to the separate-zone case. Later on, it can also be resumed from the suspension and continue for charging. We assume that the time overhead incurred by charging mode switching is negligible compared with the long charging time of electric vehicles.

B. Model Differentiation

Model Categorization. Real-time task models can be classified based on different metrics such as flexibility on parallelism [35] and abstraction levels [36]. This paper uses a categorization according to the relation between the gained utility and the amount of finished workload (charging demand here). Thus, there are three categories: deterministic, metered and hybrid task models. By the deterministic model, the utility is gained (and thus the customer pays) only if the charging demand of a charging task is fully fulfilled, otherwise the gained utility is zero (and the customer pays nothing) [7], [9], [37]. This model follows the classical all-or-nothing setting for hard real-time tasks. The metered task model allows the gained utility (and payment) to be a strictly increasing function of the amount of finished workload. The hybrid task model can be seen as a combination of the above two models. That is, each hybrid task consists of two subtasks. One uses the deterministic model, i.e., the utility of the subtask is gained only if it is fully completed. The other uses the metered model, i.e., the gained utility of the subtask increases as more workload is finished. This hybrid model is closely related to the task model adopted in the imprecise computation literature [38].

Model Suitability. EV charging loads in different application scenarios, such as home charging and park-and-charge, have different features about the temporality and user satisfaction. Thus, it needs different task models (with respect to the above categorization) to capture the distinct features of different kinds of charging loads. However, existing works that use event-driven scheduling mainly confine to the deterministic model regardless of the application scenarios, e.g., [7], [9], [37]. The deterministic model is not suitable to the park-and-charge scenario due to reasons as follows.

With the deterministic task model, customers may intend to submit stringent charging requirements. The system either accepts them and risks at economic loss due to the potential unfulfilled requirements, or simply rejects them and much dissatisfies these customers. This dilemma may also happen if with the hybrid task model. Thus, both deterministic and hybrid models are not suitable for park-and-charge. By contrast, the metered model is compatible with such strict requirements. Further, it will not happen that even though there are enough parking spots, the system rejects electric vehicles only due to their tough requirements. Hence, the metered task model is more suitable to characterize the charging load in park-and-charge systems. To be specific, the task model proposed in the previous subsection belongs to the category of metered model, and thus well fits to the park-and-charge scenario.

As mentioned above, existing works, e.g., [7], [9], [37], use real-time scheduling methods such as EDF and LLF, but they confine to the deterministic model. One key question is how these methods perform when adapted to the metered model. The following sections answer this question and further analyze more and broader event-driven scheduling methods.

C. Problem Statement

This work studies the on-line charging scheduling problem with the objective of maximizing the social welfare. A charging

[1]No matter what market mechanisms are adopted, the system needs to schedule charging tasks after deciding the energy price. This scheduling step is very important to satisfy customers and is the right focus of this paper.

schedule is to decide when to charge which vehicles. The social welfare is defined as the sum utility of electric vehicles. This charging scheduling problem pursues maximized social welfare, which is different from that of hard real-time scheduling problems whose goal is to maximize the number of tasks meeting their deadlines. Electric vehicles arrive in an on-line manner, i.e., the system knows no details of the charging tasks before the vehicles' arrival. We assume that after arrival, all parameters of a charging task are known to the system. On-line algorithms need to make scheduling decisions based only on the parameters of electric vehicles that have already arrived.

III. Algorithm Analysis Based on Competitive Analysis

In this section, we present the theoretical results based on competitive analysis. We first demonstrate the non-constant and constant competitive ratios for EDF and HUF (highest-utility first) respectively, and then provide a much stronger result, i.e., the performance upper bound of the whole class of work-conserving algorithms.

A. Analysis for EDF and LLF

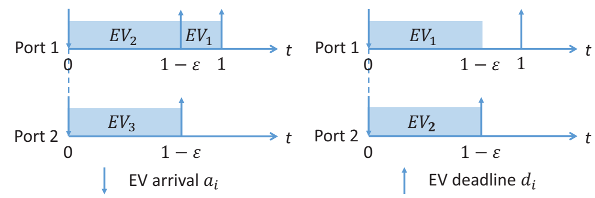
Impracticability of LLF. Earliest-deadline-first (EDF) and Least-laxity-first (LLF) are two classic temporality-based algorithms. EDF always first charges electric vehicles with the earliest deadlines (d_i); while LLF always first processes vehicles with the least laxity time ($d_i - a_i - e'_i/p$, where e'_i is the remaining charging demand). Recently, LLF has been used to schedule deferrable loads such as charging tasks [37]. However, LLF is impractical for the charging scheduling problem. The reason is that the algorithm may cause much frequent preemption or mode switching. For example, suppose there are two charging points and three vehicles. The charging power is set as $p = 1$ and the three vehicles' setting (by (a_i, d_i, e_i, u_i)) is: $EV_1 = (0, 1, 1, 1)$, $EV_2 = (0, 1, 1 - \varepsilon, 1)$, $EV_3 = (0, 1 + \varepsilon, 1 - \varepsilon, 1)$, where $0 < \varepsilon < 1$. By LLF, the number of mode transitions for EV_2 and EV_3 is both $O(1/\varepsilon)$. Thus, if ε is small, the number will be large. Actually, this number is unbounded, i.e., approaches infinity as $\varepsilon \to 0$.

Most electric vehicles in the current generation are equipped with Li-ion batteries. The lifetime and capacity of Li-ion batteries are significantly compromised by the frequent mode switching [39]. Although the next-generation battery for future electric vehicles would have better resilience to the mode switching, it is still dangerous to employ such an algorithm with unbounded number of mode transitions. Hence, we focus on the performance analysis for EDF in the following.

No constant competitive ratio for EDF. Competitive analysis provides the performance gap between an on-line algorithm and the off-line optimal algorithm. The performance gap is expressed by a definition of competitive ratio as follows. Note that the definition is for maximization problem, not for minimization problem such as [40].

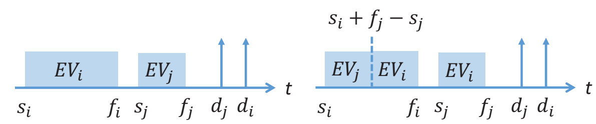
Definition 1 (Competitive ratio): An on-line algorithm A is α-competitive if $\frac{Opt(I)}{A(I)} \le \alpha$ for any feasible input instance I, where $Opt(I)$ and $A(I)$ are the social welfare gained by the off-line optimal algorithm and the algorithm A on input instance I, respectively.

Based on this definition, we demonstrate that there is no constant competitive ratio for EDF. Suppose there are two charging points and three electric vehicles. The charging power



(a) EDF (EV_1 misses deadline) (b) Opt (EV_3 misses deadline)

Fig. 3. Illustration for the example showing no constant bounds for EDF.



(a) Original schedule (b) Canonical schedule

Fig. 4. Illustration of reschedule for Lemma 1.

is set as $p = 1$ and the three vehicles' settings are as follows: $EV_1 = (0, 1, 1 - \varepsilon, 1/\varepsilon)$, $EV_2 = (0, 1 - \varepsilon, 1 - \varepsilon, 1)$, $EV_3 = (0, 1 - \varepsilon, 1 - \varepsilon, \varepsilon)$, where $0 < \varepsilon < 1$. As illustrated in Fig. 3, EV_2 and EV_3 are first charged to finish by the EDF schedule; while EV_1 and EV_2 are first charged to finish in the optimal schedule. The utility gained by $EV_1 to EV_3$ is (i) EDF: $1, 1 - \varepsilon, (1 - \varepsilon) \cdot \varepsilon$; (ii) Opt: $(1 - \varepsilon)/\varepsilon, 1 - \varepsilon, 0$. Hence, the social welfares derived by the two schedules are:

$$U(EDF) = 2 - \varepsilon^2, \quad U(Opt) = \frac{1}{\varepsilon} - \varepsilon.$$

Letting $\varepsilon \to 0$ makes the competitive ratio arbitrarily approach the positive infinity: $\lim_{\varepsilon \to 0} \frac{U(Opt)}{U(EDF)} \to +\infty$. Therefore, EDF has no constant performance bound based on competitive analysis.

The non-constant competitive ratio. Before giving this performance bound, we present several definitions and lemmas that will be used in the following theoretical proofs. They are the definition of canonical schedule, a shared property among off-line optimal schedules, and a utility mapping scheme.

Definition 2 (Canonical schedule): A schedule S is called canonical if for any two time points $t_1 < t_2$, the following is satisfied: if $z_1 = S(t_1)$, and $z_2 = S(t_2) \neq \emptyset$, then either (i) $\min_{i \in z_2}(a_i) > t_1$, or (ii) $z_1 \neq \emptyset$ and $\max_{i \in z_1}(d_i) < \min_{i \in z_2}(d_i)$, where $S(t)$ is the set of vehicles being charged in schedule S at time t.

Intuitively, Definition 2 indicates that at any time, schedule S processes either the electric vehicles with the earliest deadlines, or discard them forever. With this definition, we have Lemma 1 for the off-line optimal schedules.

Lemma 1: At least one of the off-line optimal schedules is a canonical schedule.

Proof: We prove this lemma by demonstrating that any off-line optimal schedule can be converted to a canonical schedule without affecting its optimality. Consider an off-line optimal schedule S, and EV_i with a later deadline and EV_j with a earlier deadline. We use s_i and f_i (s_j and f_j) to denote the starting and ending time of some consecutive portion of processing EV_i's (EV_j's) charging demand, where $s_i \ge \max(a_i, a_j), s_j \ge \max(a_i, a_j)$. Suppose that EV_i's portion is scheduled before EV_j's portion, i.e., $s_i < s_j$. As illustrated in Fig. 4, we can reschedule and make EV_j's portion processed ahead of EV_i's without changing their gained utility. For EV_j, it is scheduled earlier than the original, and thus its utility keeps unchanged. For EV_i, though it is postponed to a later time, its new ending time is still earlier than

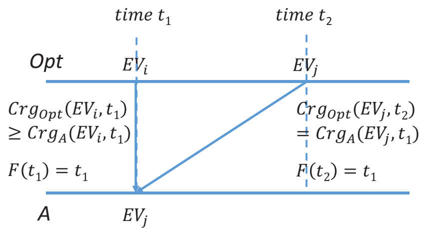


Fig. 5. The utility mapping scheme.

$\max\{f_i, f_j\}$, which is less than deadline d_i. Thus, EV_i's utility also stays unchanged. If applying the above reschedule to all non-canonical portions, we convert schedule S to a canonical schedule. Therefore, the lemma holds. ■

To bound the competitive ratio of EDF, we further employ a schedule mapping scheme proposed in [41]. We adapt this scheme to our system model and call it as utility mapping scheme. The adapted scheme is detailed as follows.

The Utility Mapping Scheme. The scheme is described by a function F: $R \to R$, which maps each time point in the off-line optimal schedule Opt to a time point in the schedule of an on-line algorithm A. We use $Crg_x(EV_i, t)$ to denote the amount of energy that has been fueled into EV_i by algorithm x by time t. For any time t, suppose EV_i is a vehicle being charged in Opt. If $Crg_{Opt}(EV_i, t) \geq Crg_A(EV_i, t)$, $F(t) = t$. Otherwise, let $F(t) = t' < t$, where t' is the maximum time that $Crg_{Opt}(EV_i, t) = Crg_A(EV_i, t)$. In both cases the unit utility is u_i. It can be seen that the utility mapped to algorithm A is equal to that of Opt. Fig. 5 illustrates this utility mapping scheme. For example, at time t_1, since $Crg_{Opt}(EV_i, t_1) \geq Crg_A(EV_i, t_1)$, the mapping is $F(t_1) = t_1$.

We define the utility ratio at any time as the sum utility mapped to A over the utility obtained by A at that time. The utility ratio at time t' in Fig. 5 is calculated by $(p{\cdot}u_i + p{\cdot}u_j)/p{\cdot}u_j$. Note that at any time, there are at most two utilities mapped from Opt to A: one is $p{\cdot}u_i$ from time t' and the other is $p{\cdot}u_j$ from time t later than t'. Hence, if we can bound the utility ratio for all time points, then this offers a bound on the competitive ratio of algorithm A.

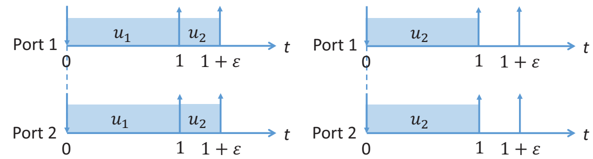
We now show the non-constant performance bound, which is dependent on the dynamic range of unit utility (see Theorem 2). This dynamic range is described by the ratio of maximum to minimum unit utility, $\beta = \max(u_i)/\min(u_i), \beta \geq 1$.

Theorem 2: EDF is β-competitive and the bound is tight.

Proof: **Bounding competitive ratio.** Suppose that at time t, $EV_1, ..., EV_M$ are the M vehicles being charged by EDF, and $EV'_1, ..., EV'_M$ are the M vehicles being charged by an off-line optimal canonical schedule Opt. Some of the EV_is may be identical to some of the EV'_is. Thus, without loss of generality, we assume $EV_i = EV'_i$ for $i \in I \subset \{1, ..., M\}$. We discuss the two individual cases (whether vehicles belong to set I) as follows.

Case 1 ($i \in I$): The utility of EV'_i by Opt can be mapped to EDF at time t at most once. The reason is that according to the utility mapping scheme described above, the EV'_i utility is either mapped to time t or a time before t. The total utility mapped in this case is at most $\sum_{i \in I} p{\cdot}u_i$.

Case 2 ($i \in \{1, ..., M\} \setminus I$): First, also according to the scheme, the utility of the set of EV'_i is mapped to EDF at most once. The sum utility is at most $\sum_{i \in \{1,...,M\} \setminus I} p{\cdot}u'_i$. Second, the set of EV_i is not being charged in Opt at time



(a) EDF schedule (b) Opt schedule

Fig. 6. Illustration of tightness for Theorem 2. Text in rectangles is unit utility.

t, and they will not be mapped to time t from a later time. The reason is that EDF always first schedules vehicles with earliest deadlines, and thus it must hold that $d_i < \min_i(d'_i)$. Moreover, since Opt is canonical, these EV_is will be dropped by Opt forever. Thus, the mapped utility is zero.

Note that the utility obtained by the EDF schedule at time t is $\sum_{i \in \{1,...,M\}} p{\cdot}u_i$. Hence, the competitive ratio α of EDF is bounded by

$$\alpha \leq \frac{\sum_{i \in I} p{\cdot}u_i + \sum_{i \in \{1,...,M\} \setminus I} p{\cdot}u'_i}{\sum_{i \in \{1,...,M\}} p{\cdot}u_i} \leq \frac{M \cdot \overline{u}}{M \cdot \underline{u}} = \beta, \quad (1)$$

where $\overline{u} = \max(u_i)$ and $\underline{u} = \min(u_i)$. EDF is β-competitive.

Tightness of the bound. Consider a system with two charging points and four vehicles for charging. Charging power is set as 1. Two vehicles are set as $(0, 1, 1, u_1)$, and the other two are set as $(0, 1 + \varepsilon, 1, u_2)$, where $u_1 < u_2$. For EDF, as depicted in Fig. 6(a), the gained utility of the first two vehicles is $2u_1$, and that of the other two is $2\varepsilon \cdot u_2$. For Opt, as illustrated in Fig. 6(a), they are 0 and $2u_2$ respectively. Thus, the competitive ratio $\alpha = 2u_2/(2u_1 + 2\varepsilon \cdot u_2)$, which approaches to $\beta = u_2/u_1$ as $\varepsilon \to 0$. Therefore, the bound is tight and the theorem holds. ■

Theorem 2 confirms that the competitive ratio of EDF is non-constant. Furthermore, the performance gap between EDF and the optimal schedule is unpredictable if the dynamic range of unit utility is unbounded. Thus, an on-line algorithm with constant competitive ratio may be more appealing. We will analyze an algorithm in first-fit style and demonstrate its constant competitiveness in Section III-B.

We have the following corollary based on Theorem 2.

Corollary 3: EDF is optimal if all electric vehicles have the same unit utility, i.e., $\beta = 1$.

It is well-known that as to global multiprocessor (or multiple-charging-point here) scheduling, EDF is not optimal for meeting deadlines. By contrast, this corollary indicates that if all customers can be equally satisfied, EDF is optimal with respect to maximizing social welfare or optimizing user satisfaction. In other words, EDF can process charging demand as much as the off-line optimal algorithm does.

B. A 2-Competitive Scheduling Algorithm

We analyze a first-fit style algorithm called highest-utility-first (HUF). The algorithm always first charges EVs with the highest unit utilities. As we know, the algorithm is similar to the efficiency (profit to weight ratio) first algorithm of the knapsack problem [42], which first packs items with highest efficiencies. However, our system model is different from that of the knapsack problem. The difference includes the metered model, deadline and preemption. Hence, the analysis for the efficiency first algorithm can not be applied to our problem. To bound the competitive ratio of HUF, we also leverage the utility mapping scheme described above. Using a similar proof skeleton to Theorem 2, we prove a 2-competitiveness for HUF, i.e., HUF always gives a welfare at least half of the optimal.

Theorem 4: HUF is 2-competitive and the bound is tight.

Proof: **Bounding competitive ratio.** Suppose that at time t, $EV_1, ..., EV_M$ are the M vehicles being charged by HUF with $u_1 \geq u_2 \geq ... \geq u_M$, and $EV_1', ..., EV_M'$ are the M vehicles being processed by an off-line optimal schedule Opt. Some EV_is may be identical to some EV_i's. Thus, without loss of generality, we assume $EV_i = EV_i'$ for $i \in I \subset \{1, ..., M\}$. We discuss the two individual cases as follows.

Case 1 ($i \in I$): The utility of EV_i' by Opt can be mapped to HUF at time t at most once, since the EV_i' utility is either mapped to time t or a time before t. Thus, the total utility mapped is at most $\sum_{i \in I} p \cdot u_i$.

Case 2 ($i \in \{1, ..., M\} \setminus I$): First, according to the scheme, the utility of the set of EV_i' either is not mapped to HUF at time t, or if it is, the following condition holds. We must have $p \cdot u_i' \leq p \cdot u_M$ because these vehicles are not finished for charging and not chosen by HUF. The utility (of EV_i') mapped is at most $(M - |I|) \cdot p \cdot u_M$, where $|I|$ is the number of vehicles in set I. Second, the set of EV_i is not being charged in Opt at time t, and they may be mapped to time t from a later time in Opt. Thus, the utility (of EV_i) mapped is at most $\sum_{i \in \{1,...,M\} \setminus I} p \cdot u_i$.

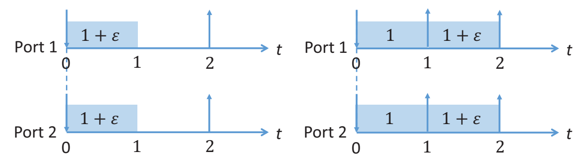
Note that the utility obtained by the HUF schedule at time t is $\sum_{i \in \{1,...,M\}} p \cdot u_i$. Hence, the competitive ratio (α) of HUF is bounded by

$$\begin{aligned} \alpha &\leq \frac{\sum_{i \in I} p \cdot u_i + \sum_{i \in \{1,...,M\} \setminus I} p \cdot u_i + (M - |I|) \cdot p \cdot u_M}{\sum_{i \in \{1,...,M\}} p \cdot u_i} \\ &= \frac{\sum_{i \in \{1,...,M\}} p \cdot u_i + (M - |I|) \cdot p \cdot u_M}{\sum_{i \in \{1,...,M\}} p \cdot u_i} \\ &\leq \frac{\sum_{i \in \{1,...,M\}} p \cdot u_i + M \cdot p \cdot u_M}{\sum_{i \in \{1,...,M\}} p \cdot u_i} \quad (2) \\ &\leq \frac{\sum_{i \in \{1,...,M\}} p \cdot u_i + \sum_{i \in \{1,...,M\}} p \cdot u_i}{\sum_{i \in \{1,...,M\}} p \cdot u_i} = 2. \quad (3) \end{aligned}$$

Eqn. (2) is true because of that the set I may be empty, i.e., $|I|$ may be equal to zero. As mentioned above, we must have $u_1 \geq ... \geq u_M$ according to HUF. Thus, $M \cdot p \cdot u_M \leq \sum_{i \in \{1,...,M\}} p \cdot u_i$ and thus we have Eqn. (3).

Tightness of the bound. Consider a system with two charging points and four vehicles for charging. Charging power is set as 1. Two vehicles are set as $(0, 2, 1, 1+\varepsilon)$, and the other two are set as $(0, 1, 1, 1)$, where $\varepsilon > 0$. As shown in Fig. 7(a), HUF only gains the utility of the first two vehicles and none for the other two due to deadline miss. As depicted in Fig. 7(b), the off-line optimal algorithm obtains all four vehicles' utility. The competitive ratio $\alpha = 4/(2 + 2\varepsilon)$, which approaches to 2 as $\varepsilon \to 0$. Hence, the bound is tight and the theorem holds. ■

As defined in Section II-A, the unit utility of a charging task is a piecewise function and each subtask corresponds to one piece. That is, the unit utility varies between subtasks and is fixed for each individual subtask. Thus, by HUF, subtasks being served always have higher unit utilities than do suspended charging (sub)tasks. The former subtasks can never be preempted by the latter ones. Hence, this utility definition guarantees that HUF will not behave in the same way as LLF with unbounded number of preemptions or mode transitions.



(a) HUF schedule (b) Opt schedule

Fig. 7. Illustration of tightness for Theorem 4. Text in rectangles is unit utility.

C. Bounding All Work-Conserving Schedules

We present a stronger result through showing that any work-conserving algorithm is bounded. It is worth to note that the performance bound is applicable to the whole class of work-conserving algorithms, not just to a specific algorithm. In the following, we first define a work-conserving schedule and then analyze the upper bound.

Definition 3 (Work-conserving schedule): A schedule S is called work-conserving if the following is satisfied: at any time t, if there are unfinished vehicles with $a_i \leq t < d_i$, then $S(t) \neq \emptyset$, where $S(t)$ is the set of vehicles being charged in schedule S at time t.

Definition 3 means that the system can not be idle if there are still unfinished and unexpired charging demands. Note that the algorithms discussed in this paper are all work-conserving. Moreover, the optimal off-line schedules must be work-conserving, otherwise a larger social welfare can be obtained by rescheduling vehicles to fill the idleness.

Theorem 5: Any work-conserving algorithm is $(1 + \beta)$-competitive, i.e., $1 + \beta$ is the upper bound.

Proof: The proof is similar to that of Theorem 2. Also suppose that at time t, $EV_1, ..., EV_M$ are the M vehicles in a work-conserving schedule S, and $EV_1', ..., EV_M'$ are in Opt. The difference lies in Case 2. The set of EV_i that is not in Opt but in S, may be mapped to time t from a later time. Hence, the competitive ratio of schedule S is

$$\begin{aligned} \alpha &\leq \frac{\sum_{i \in I} p \cdot u_i + \sum_{i \in \{1,...,M\} \setminus I} p \cdot u_i' + \sum_{i \in \{1,...,M\} \setminus I} p \cdot u_i}{\sum_{i \in \{1,...,M\}} p \cdot u_i} \\ &= 1 + \frac{\sum_{i \in \{1,...,M\} \setminus I} p \cdot u_i'}{\sum_{i \in \{1,...,M\}} p \cdot u_i} \leq 1 + \frac{M \cdot \overline{u}}{M \cdot \underline{u}} = 1 + \beta, \quad (4) \end{aligned}$$

where $\overline{u} = \max(u_i)$ and $\underline{u} = \min(u_i)$. The theorem holds. ■

The ratio $1+\beta$ is a performance upper bound for the whole class of work-conserving algorithms. EDF and HUF are also work-conserving algorithms and they have better performance bounds. For example, the competitive ratio of HUF is $2 \leq 1 + \beta$. Based on Theorem 5, we can obtain the performance bound for a batch of on-line algorithms such as first-come-first-serve (FCFS) and shortest-job-first (SJF). FCFS always first processes electric vehicles with the earliest arrival times; while SJF always charges vehicles with the least charging demands.

Corollary 6: Both FCFS and SJF are $(1 + \beta)$-competitive and the bound is tight.

Proof: The bound directly follows Theorem 5 and we only need to prove the tightness. Consider a system with two charging points and four vehicles for charging. The charging power is set as 1. For FCFS, two electric vehicles are set as $(0, 2, 1, u_1)$, and the other two are set as $(\varepsilon, 1, 1 - \varepsilon, u_2)$, where $u_1 < u_2$. The gap between Opt and FCFS is $\alpha = (2 \cdot u_1 + 2 \cdot u_2)/2 \cdot u_1 = 1 + \beta$. For SJF, two vehicles are set as $(0, 2, 1, u_1)$, and the other two are set as $(0, 1 + \varepsilon, 1 + \varepsilon, u_2)$, where $u_1 < u_2$. The gap between Opt and SJF is $\alpha = (2(1 +$

TABLE I. SUMMARIZATION OF THEORETICAL RESULTS USING COMPETITIVE ANALYSIS.

Algorithm	EDF	HUF	FCFS/SJF	work-conserving
Bound	β	2	$1+\beta$	$1+\beta$
Tight	Yes	Yes	Yes	No

$\varepsilon) \cdot u_1 + 2(1-\varepsilon) \cdot u_2)/(2 \cdot u_1 + 2\varepsilon \cdot u_2)$, which approaches to $1+\beta$ as ε goes to zero. Therefore, the corollary holds. ■

Table I summarizes the theoretical results based on competitive analysis in this section. As mentioned above, multiple charging points can be abstracted as the multi-processer case as to the scheduling theory. Thus from a technical perspective, the theoretical results in this section can be seen as (i) a generalization to [41], which only focuses on the uni-processor case, and (ii) an extension to [22], which solely studies competitive analysis for HUF.

IV. Algorithm Analysis Based on Resource Augmentation

In this section, we leverage an analysis technique called resource augmentation for performance analysis. Resource augmentation as a method for analyzing on-line algorithms, first appears in [43], [44]. An on-line algorithm is given more resource than the optimal off-line schedule with which it is compared. The idea is to analyze how much additional resource the on-line algorithm is needed to achieve the optimality. For a park-and-charge system, the additional resource means either higher charging power or more charging points. We first present the analysis for EDF and propose the results for the whole class of work-conserving algorithms. In the following theorems, p and M denote the charging power and the number of charging points for an optimal off-line schedule respectively.

A. Analysis for EDF

In Section III-A, we discuss EDF using competitive analysis and prove its non-constant competitive ratio. Because constant bounds are more appealing, one pending question is whether EDF has a constant performance bound if using other analysis techniques. We thus turn to resource augmentation. The authors in [22] adopts resource augmentation to analyze EDF for the uni-processor (or single charging point) case. We make a significant generalization to that and deal with the case of multiple charging points. We demonstrate that EDF also has no constant performance bound according to the following two lemmas. This result partly answers the pending question through eliminating one more analysis technique.

Lemma 7: EDF is optimal with charging power $\beta \cdot p$.

Proof: **Bounding augmented resource.** The proof is similar to that of Theorem 2. Thus, we only highlight the difference as follows. Suppose that EDF is with charging power of $x \cdot p$ and *Opt* is with power p. The utility by EDF is thus $x \cdot \sum_{i\in\{1,\ldots,M\}} p{\cdot}u_i$. The mapped utilities for Case 1 and Case 2 keep unchanged, which are $\sum_{i\in I} p{\cdot}u_i$ and $\sum_{i\in\{1,\ldots,M\}\setminus I} p{\cdot}u_i'$ respectively. Thus, we have

$$\frac{\sum_{i\in I} p{\cdot}u_i + \sum_{i\in\{1,\ldots,M\}\setminus I} p{\cdot}u_i'}{x \cdot \sum_{i\in\{1,\ldots,M\}} p{\cdot}u_i} \le \frac{M \cdot \overline{u}}{x \cdot M \cdot \underline{u}} = \frac{\beta}{x}.$$

Hence, to make EDF is 1-competitive (i.e., optimal), we let the above equation less than or equal to 1. That is, if $x = \beta$, i.e., the charging power is augmented to $\beta \cdot p$, EDF is optimal.

Tightness of the bound. We prove this tightness using the same example as that in Theorem 2. For EDF with augmented resource, as depicted in Fig. 8(a), the gained utility of the first two vehicles is $2u_1$ and that of the other two is $2u_2 \cdot (1+\varepsilon-u_1/u_2)$ respectively. For Opt, as shown in Fig. 8(b), they are 0 and $2u_2$ respectively. Thus, the competitive ratio is $\alpha = 2u_2/(2u_2 + 2\varepsilon \cdot u_2)$, which approaches to 1 as $\varepsilon \to 0$. The lemma holds. ■

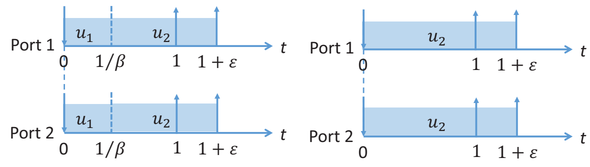


Fig. 8. Illustration of tightness for Lemma 7. Text in rectangles is unit utility.

Lemma 8: EDF is optimal with $\lceil M \cdot \beta \rceil$ charging points.

Proof: **Bounding augmented resource.** The proof is also similar to that of Theorem 2 and we highlight the difference. Suppose that EDF is with x charging points and *Opt* is with M charging points. The utility by EDF is thus $\sum_{i\in\{1,\ldots,x\}} p{\cdot}u_i$. The mapped utilities for Case 1 and Case 2 keep unchanged. Thus, we have

$$\frac{\sum_{i\in I} p{\cdot}u_i + \sum_{i\in\{1,\ldots,M\}\setminus I} p{\cdot}u_i'}{\sum_{i\in\{1,\ldots,x\}} p{\cdot}u_i} \le \frac{M \cdot \overline{u}}{x \cdot \underline{u}} = \frac{M \cdot \beta}{x}.$$

Hence, to make EDF is 1-competitive, we let the above equation less than or equal to 1. That is, if $x = M \cdot \beta$, i.e., the number of charging points is augmented to $\lceil M \cdot \beta \rceil$ (the number is integral), EDF is optimal.

Tightness of the bound. This tightness can be proved by constructing two different schedules as follows. One is *Opt*, where all scheduled vehicles have the highest unit utilities. The other is the EDF with more charging points, where all scheduled vehicles have the lowest unit utilities. For example, consider a system with two charging points. Suppose that there are two vehicles with unit utility u_2 and later deadlines, and 2β vehicles with u_1 and earlier deadlines, where $u_2 > u_1$ and $\beta = u_2/u_1$. For Opt, the welfare is $2 \cdot u_2$. For EDF with 2β charging points, the welfare is $2\beta \cdot u_1$. It is easy to verify that the competitive ratio is 1. Hence, the lemma holds. ■

B. Analysis for Work-Conserving Schedules

Using similar proofs to those in the above lemmas, we prove that HUF has a constant bound based on resource augmentation, as illustrated in Lemma 9. We further have a stronger result, Theorem 10, which is an upper bound of augmented resource for the whole class of work-conserving algorithms. We omit their proofs to avoid repetition.

Lemma 9: HUF is optimal with charging power $2p$ or with $2M$ charging points.

Theorem 10: Any work-conserving algorithm is optimal with charging power $(1+\beta) \cdot p$ or $\lceil M \cdot (1+\beta) \rceil$ charging points.

Corollary 11: Both FCFS and SJF are optimal with charging power $(1+\beta) \cdot p$ or with $\lceil M \cdot (1+\beta) \rceil$ charging points.

Table II summarizes the theoretical results based on resource augmentation in this section. One application of these results is when a park-and-charge system plans to upgrade its charging infrastructure. These results can answer questions such as how much charging power and how many charging points need to be added in order to satisfy customers to a

TABLE II. SUMMARIZATION OF THEORETICAL RESULTS BASED ON RESOURCE AUGMENTATION. p (M) DENOTES THE CHARGING POWER (THE NUMBER OF CHARGING POINTS) FOR AN OPTIMAL OFF-LINE SCHEDULE.

Algorithm	EDF	HUF	FCFS / SJF	work-conserving
Charging Power	$\beta \cdot p$	$2 \cdot p$	$(1+\beta) \cdot p$	$(1+\beta) \cdot p$
Charging Point No.	$\lceil \beta \cdot M \rceil$	$2 \cdot M$	$\lceil (1+\beta) \cdot M \rceil$	$\lceil (1+\beta) \cdot M \rceil$
Tight	Yes	Yes	Yes	No

certain degree. From a technical perspective, the theoretical results in this section can be seen as generalization to that of the uni-processor case studied in [22], [41]. Further, as mentioned above, our work adopts the metered task model and has a goal of welfare maximization. This differentiates it from those existing works such as [43], [44] that use deterministic task model and optimize pure temporal metrics such as response time and deadline miss.

V. EVALUATION

We evaluate the performance of four different algorithms including HUF, EDF, FCFS and SJF. As mentioned in Section III-A, LLF is impractical for park-and-charge system and thus is excluded in our evaluation. We carry out comparisons between them and the off-line optimal solution (Opt). To derive the optimal, we formulate an integer linear program for the charging scheduling problem, which is then solved using IBM ILOG CPLEX [45]. We study the performance of these algorithms under different ranges of unit utility (i.e., different values of β), and different scales of charging resource (i.e., different values of p and M), different charging load of vehicles (i.e., different numbers of EVs and charging demand).

A. Experimental Setup

We consider a park-and-charge system with M charging points and each charging point supports a charging power of p kw. We compare the social welfare in a time duration of $T = 24$h between the above algorithms and Opt. To make the integer linear program solvable, the time unit is set as $0.1h$ and all parameters except unit utility are set to be integral. Note that there is no such limitation for the algorithms discussed in this paper. During the 240 time units, there are N electric vehicles that come to the system for charging. The arrival time a_i is uniformly generated from $[1, 240]$ time unit. The deadline d_i, charging demand e_i and unit utility u_i are chosen from uniform distributions in $[10, D]$ time unit, $[5, C]$ kWh and $[1, \beta]$, respectively. Note that each figure in the following plots the results that are already normalized to the maximum value in the corresponding setting. The following shows the simulation results as well as the corresponding result analysis.

B. Experimental Results

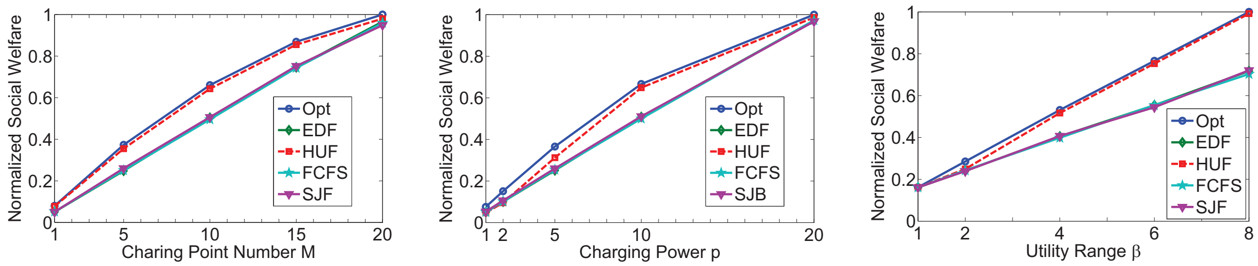
1) Welfare Comparison: Fig. 9 and Fig. 10 demonstrate the welfare comparison under different settings between the four algorithms and the off-line optimal result. First, HUF significantly outperforms the other three algorithms on welfare maximization across nearly all settings. The reason is that HUF always first schedules electric vehicles with larger unit utilities, while other three algorithms schedule vehicles based on their temporal characteristics. Furthermore, HUF performs nearly as well as Opt across all of the settings. In practice, the performance gap between HUF and Opt is much smaller than the theoretical bound, and so do the other three algorithms. For example, when $\beta = 8$ (in Fig. 9(c)), the theoretical bound of EDF is 8 according to Theorem 2, but the experimental performance gap between EDF and Opt is about 1.4. Another observation is that EDF, FCFS and SJF have nearly the same performance (i.e., the three curves overlap with each other), though their theoretical performance bounds are quite different (β- and $1 + \beta$-competitiveness). This observation infers that only differentiating vehicles' temporal characteristics helps little on welfare maximization with the metered charging model, when the unit utility range β is large.

2) Sensitivity Analysis: **Impact of charging point number.** Fig. 9(a) plots social welfare varying with the number of charging points. First, the welfare of all four algorithms as well as Opt increases as the number of charging points rises. This observation follows the intuition that more charging points can process more vehicles and thus results in higher welfare. Second, an interesting observation is that as the charging point number grows, the difference on welfare between Opt (or HUF) and the other three algorithms first increases and then decreases. The reason is two-fold. On one hand, all algorithms and Opt can process only a small amount of vehicles when charging points are scarce (e.g., $M = 1$), and they can finish almost all charging demands when charging points become abundant (e.g., $M = 20$). On the other hand, in-between (e.g., $M = 5 \ to \ 15$) is when much difference occurs to the set of vehicles that these algorithms can schedule, and so does the welfare. Third, the augmented resource (charging points) in practice is less than the theoretical bound. For example, the bound for EDF is $\lceil \beta \cdot M \rceil$. However, the EDF value at $M = 20$ is much larger than the Opt value at $M = 5$. Similar remarks can be also made for HUF, FCFS and SJF.

Impact of charging power. Fig. 9(b) shows welfare varying with the charging power. For this setting, we have similar observations and analysis to those for Fig. 9(a). We thus only outline them as follows. First, the welfare of all four algorithms and Opt increases as the charging power rises. Second, the difference on welfare between Opt (or HUF) and the other three algorithms first goes up and then declines. Third, the augmented resource (charging power) in practice is less than the theoretical bound. For instance, the EDF value at $p = 20$ is much larger than the Opt value at $p = 5$, where charging power grows 4 times.

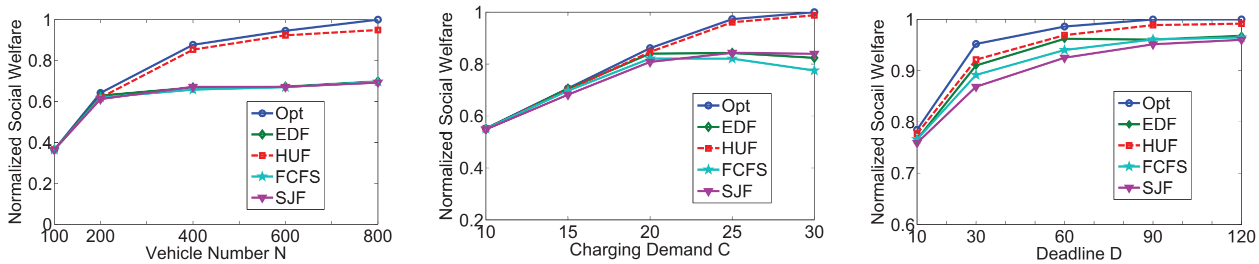
Impact of unit utility range. Fig. 9(c) demonstrates welfare varying with the range of unit utility. As the utility range grows, the welfare of all four algorithms and Opt increases, and the performance gap becomes wider and wider. As to flat unit utility (i.e., $\beta = 1$), the welfare derived by EDF is equal to the Opt value. This observation validates Corollary 3. The other three algorithms also perform nearly the same as Opt. This is because that since the unit utility is flat, the resulting gap between them and Opt is only caused by temporality, which is thus rather small. When the range of unit utility is $\beta = 8$, the performance difference between Opt and EDF, FCFS, or SJF becomes to about 40%. By contrast, HUF still performs nearly as well as Opt. This observation indicates the robustness of HUF on welfare maximization.

Impact of vehicle number. Fig. 10(a) plots welfare varying with the number of electric vehicles. First, the welfare of all four algorithms and Opt increases as the number of vehicles rises. This is because there are more vehicles scheduled for charging. However, the increasing rate on welfare decreases. For example, as to Opt and HUF, the welfare increases more than 75% from $N = 100$ to $N = 200$; while it is less than 6% from $N = 600$ to $N = 800$. For the other three algorithms,



(a) $p = 10, \beta = 4$ (b) $M = 10, \beta = 4$ (c) $M = 10, p = 10$

Fig. 9. Experimental results for varying charging point number (M), charging power (p) and unit utility range (β). Parameters on vehicle number, charging demand and deadline are set as $N = 400, C = 20, D = 60$.



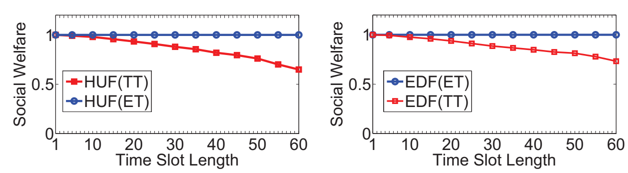
(a) $C = 20, D = 60, \beta = 4$ (b) $N = 200, D = 60, \beta = 8$ (c) $N = 200, C = 20, \beta = 8$

Fig. 10. Experimental results for varying parameters on vehicle number (N), charging demand (C) and deadline (D). Charging point number and charging power are set as $M = 10, p = 10$.

there is nearly no change on welfare from $N = 200$ to 800. The reason is as follows. When the number of vehicles is small, the system is lightly loaded and all charging demand can be fulfilled. As the number grows, the system's charging load becomes heavier and heavier. After it reaches the load limit that an algorithm can schedule (e.g., $N = 200$ for EDF), the welfare by the algorithm stays nearly unchanged. Second, because of the same reason, the difference on welfare between Opt or HUF and the other three algorithms grows as the number of vehicles rises, especially when $N \geq 200$. This observation indicates that HUF is more suitable to systems that tend to be overloaded. As to the charging service, the park-and-charge system is such a system. In the real-life scenario, the charging point or power supply in one system is limited compared with the parking space and is an even scarcer resource compared with the scale of electric vehicles.

Impact of charging demand. Fig. 10(b) demonstrates welfare varying with the charging demand of vehicles. In reality, it seldom happens that all vehicles have the same charging demand. Thus for the charging demand range $[5, C]$ kWh, we set that C varies from 10 kWh. For this setting, we can make similar observations and analysis to those for Fig. 10(a). For example, the welfare by Opt and HUF increases as the charging demand rises, and the increasing rate decreases. Instead of repeating them, we only highlight a different observation as follows. The social welfare by EDF and FCFS even declines as the charging demand grows, which is rather counter-intuitive. The reason for EDF is that vehicles with earlier deadlines may be with lower unit utility. Increasing the charging demand of these vehicles may make the system schedule less vehicles with higher unit utility. Similarly, the reason for FCFS is that increasing the charging demand of vehicles with earlier arrival time may reduce the serving time for vehicles with higher unit utility. This observation reveals the performance anomaly of the temporality-based methods on welfare maximization.

Impact of deadline. Fig. 10(c) shows welfare varying with the deadline of vehicles. First, the welfare of all four algorithms and Opt increases as the deadline rises. The reason is that longer deadlines make more charging demand be processed and thus results in higher welfare. Second, the increasing rate on welfare decreases as the deadline grows. For example, the welfare by HUF grows about 10% from $D = 10$ to $D = 30$; while it stays nearly uncharged from $D = 90$ to $D = 120$. This is because that the welfare benefits little from deadline increasing after the deadline is large enough to make all charging demand fully satisfied.



(a) Comparison on HUF (b) Comparison on EDF

Fig. 11. Comparisons about event-driven and time-driven versions of HUF and EDF. ET: event-driven; TT: time-driven.

3) Event-driven vs. Time-driven: The above comparisons (Section V-B1 and V-B2) confine to event-driven algorithms. Here shows the comparison between event-driven and time-driven policies. Time-driven policy has been widely used for general EV charging problems in existing works such as [14]–[19], but it is inadequate for the park-and-charge scenario compared to event-drive policy. This fact is validated by Fig. 11, which depicts exemplary comparisons using event-driven and time-driven versions of HUF and EDF. We can see that the event-driven versions perform better than the time-driven versions. Their difference becomes larger as the time slot length increases. The reason is that by time-driven policy, the scheduler is activated only at the beginning of each time slot. If a charging task arrives within a time slot, it needs to wait for being scheduled until the beginning of next time slot. The longer the time slot is, the more waiting time the task would have. For the extreme case that a charging task arrives just after a time slot begins, it will delayed for an entire time slot. Thus time-driven policy can much compromise the social welfare as to park-and-charge.

VI. Conclusion

This paper studies a park-and-charge system and focuses on the on-line charging scheduling problem for maximizing social welfare or EV user satisfaction. We propose to adopt the metered model to characterize charging tasks and employ event-driven methods to schedule charging tasks. Our goal is to perform both theoretical and experimental analysis for event-driven algorithms adapted to the metered model. Theoretical results show the varying performance bound of temporality-based algorithms such as EDF, FCFS, and SJF, and the constant performance bound of the value-based algorithm, HUF. Simulation results further demonstrate the near-optimality and performance consistency of HUF, and the sub-optimality and performance anomaly of the temporality-based algorithms. These results indicate that HUF has better robustness than those temporality-based algorithms do in terms of maximizing social welfare. Hence, adopting HUF in park-and-charge systems can provide better charging service to customers and thus make them better satisfied.

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